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Abstract Logic or the Science of Modality

BY

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I

THE logician recognizes the following propositional forms as necessary and sufficient for the expression of any truth:

The *categorical* forms:

$A(ab)$ = All a is b
$E(ab)$ = No a is b
$U(ab)$ = All non-a is b
$O(ab)$ = Not all a is b—Some a is not b
$I(ab)$ = Some a is b
$V(ab)$ = Not all non-a is b—Some non-a is not b

The *hypothetical* form:

X implies $Y = X \angle Y$
X does not imply $Y = (X \angle Y)'$

The *conjunctive* form:

X and $Y = X \cdot Y$

The *disjunctive* form:

X or $Y = X + Y$

Terms or classes will be represented by the letters $a, b, c, \ldots$

propositions by the letters $p, q, r, \ldots, x, y, z, \ldots$ The null-proposition is one that satisfies the implication

$$X \angle X'$$

and is represented by the symbol o. Its negative the one-proposition will therefore be defined as satisfying the implication

$$X' \angle X$$

It will be represented by the symbol 1.

A proposition whose truth-value, zero or one, is in general dependent on the meaning of the terms that enter into it, is said to be variable. It will be represented as a function of its terms, $p(a, b, c, \ldots)$. If $p, q, r, \ldots$ be propositional functions, then the expression $|p, q, r, \ldots|$ may be provisionally interpreted as the assertion, true or false,

There exist specific meanings of the terms that enter into $p, q, r, \ldots$ that make $p, q, r, \ldots$ simultaneously true.

The expression $|p, q, r, \ldots|$ is called the modal of the n elements $p, q, r, \ldots$ It is said to be of the nth. degree and, if none of its elements contain the modal symbol, to be of the first order. When we wish to indicate that the modal operator is repeated n times, we shall write:

$$|p, q, r, \ldots|_n$$

Thus

$$|p|_2 = ||p||, \qquad |p|_3 = |||p|||, \qquad |p| = ||p|'|'_0$$

The relation of any variable p to any modal function of p, that is, all the simple properties of the modal function, follow at once from the following assumptions:

Definition:

$$|p| = (p \angle o)'$$

Principle:

$$p \angle ||p|'|'_n \cdot \angle \cdot p \angle ||p|'|'_{n+1}$$

Postulate:

$$p \angle (p \angle o)'$$

Theorem:

$$p \angle ||p|'|'_n$$

If we symbolize p and q by "lengths" and $p \angle q$ as the length p contained in the length q, these properties may be exhibited in the table set forth below.

Substituting $n = 0, 1, 2, 3$, in our last theorem we obtain,

(1) $P \angle P|$

(2) $P \angle P|'|'$

(3) $P \angle P|'||'$

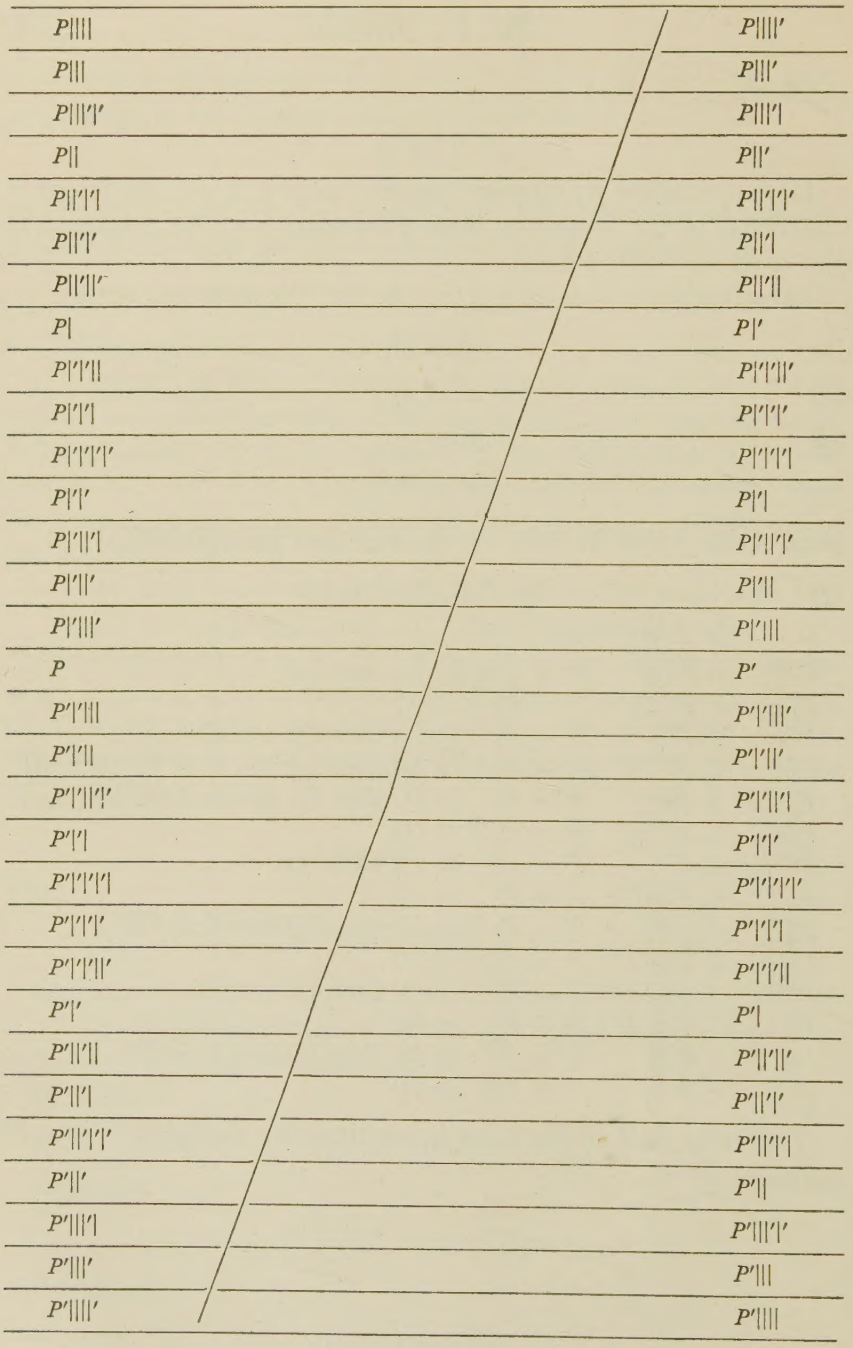
(4) $P \angle P|'|||'$

from which follow at once the theorems set down below by

(5) $(p \angle q) \angle (q' \angle p')$

P	$\angle P\|'\|\|'$	(4)
$P\|'\|\|'$	$\angle P\|'\|'$	$P = P\|'\|\|$ in (1) then (5)
$P\|'\|'$	$\angle P\|'\|'\|$	$P = P\|'\|'$ in (1)
$P\|'\|'\|$	$\angle P\|'\|'$	$P = P\|$ in (2) then (5) and $P = P\|'$
$P\|'\|'$	$\angle P\|'\|'\|'\|'$	$P = P\|'$ in (2) then (5) and $P = P\|'$ then (5)
$P\|'\|'\|'\|'$	$\angle P\|'\|'\|$	$P = P\|'$ in (1) then (5) and $P = P\|'\|'$
$P\|'\|'\|$	$\angle P\|'\|'\|\|$	$P = P\|'\|'\|$ in (1)
$P\|'\|'\|\|$	$\angle P\|$	$P = P\|'$ in (3) then (5)
$P\|$	$\angle P\|\|'\|\|'$	$P = P\|$ in (3)
$P\|\|'\|\|'$	$\angle P\|\|'\|'$	$P = P\|$ in (1) then (5) and $P = P\|\|'$
$P\|\|'\|'$	$\angle P\|\|'\|'\|$	$P = P\|\|'\|'$ in (1)
$P\|\|'\|'\|$	$\angle P\|\|$	$P = P\|\|'$ in (2) then (5)
$P\|\|$	$\angle P\|\|\|'\|'$	$P = P\|\|$ in (2)
$P\|\|\|'\|'$	$\angle P\|\|\|$	$P = P\|'$ in (1) then (5) and $P = P\|\|$
$P\|\|\|$	$\angle P\|\|\|\|$	$P = P\|\|\|$ in (1)

The other half of the table is obtained by changing P into P' throughout.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The equation $|p|\cdot|p'| = 0$ has two unique solutions, zero and one. Consequently, to deny that P satisfies this equation is the same as to assert that P is a *variable*. Let P stand for the function

$$(a \angle b)\cdot\{(b \angle a) + (a \angle b')'(b' \angle a)'\}$$

where $a \angle b$, a is included in b, stands for Boolean inclusion, then

Postulate: P is not a solution of the equation

$$|p|\cdot|p'| = 0$$

Theorem:

$$(a' \angle b)\cdot\{(b \angle a') \angle (a \angle b)'(b \angle a)'\}$$

$$(a \angle b')\cdot\{(b' \angle a) + (a \angle b)'(b \angle a)'\}$$

are not solutions of the equation

$$|p|\cdot|p'| = 0$$

Moreover if P is not a solution of this equation, then P' is evidently not a solution. Accordingly,

Theorem:

$$(a \angle b)' + (b \angle a)'\cdot\{(a \angle b') + (b' \angle a)\}$$

$$(a' \angle b)' + (b \angle a')\cdot\{(a \angle b) + (b \angle a)\}$$

$$(a \angle b')' + (b' \angle a)\cdot\{(a \angle b) + (b \angle a)\}$$

are not solutions of the equation

$$|p|\cdot|p'| = 0$$

From the results of our table by means of the fundamental formula,

$$|pq| = pq + p'q'\cdot|p|\cdot|q| + |p|\cdot|q'|' + |q|\cdot|p'|'$$

we may prove the

Theorem: P is not a solution of the equations

$$p\cdot|p'| = 0, \qquad |p|\cdot p' = 0$$

for

$$|p|\cdot|p'| \angle |p\cdot|p'||, \qquad |p|\cdot|p'| \angle |p'\cdot|p||$$

Principle:

$$|p|_{n+2} \angle |p|_{n+1} \cdot \angle \cdot |p|_{n+1} \angle |p|_n$$

Theorem: P is not a solution of the equations

$$|p|'_n \cdot |p|_{n+1} = 0, \qquad |p'|'_n \cdot |p'|_{n+1} = 0.$$

Definition: A modal function containing an even number of symbols of negation is termed an affirmative mode; otherwise negative, or a mode that denies.

Definition: Lp, Mp, Np, stand for some modal function of p.

Principle: If M and N are the one affirmative, the other negative, if M is of higher order than N and if P is not a solution of the equation $M \cdot N = 0$, then P will be a solution of neither of the equations $|M'|' \cdot N = 0$ nor $M \cdot |M'| = 0$.

By means of these results it follows that the converse of all the implications recorded in the table are untrue. The induction carried as far as fourth-order modals, is sufficient to enable us to see that the results of the table may be extended to infinity in both directions without introducing any further assumption.

P is not a solution of the equation $p' \cdot |p| = 0$ and consequently, by our principle does not satisfy

$$||p|'| \cdot |p| = 0, \qquad p' \cdot ||p|'|' = 0$$

and, accordingly, does not satisfy

$$|||p|'|'|' \cdot |p| = 0 \qquad ||p|'| \cdot |||p|'|'| = 0$$
$$p' \cdot |||p|'||' = 0 \qquad |||p|'|| \cdot ||p|'|' = 0$$

and, therefore, is not a solution of

(1) $$||||p|'|'||' \cdot |p| = 0$$
(2) $$|||p|'|'|' \cdot ||||p|'|'|| = 0$$
(3) $$||||p|'|'|'|' \cdot |||p|'|'| = 0$$
(4) $$||p|'| \cdot ||||p|'|'|'|' = 0$$
(5) $$||||p|'||'|' \cdot ||p|'|' = 0$$
(6) $$|||p|'|| \cdot ||||p|'||'| = 0$$
(7) $$||||p|'||| \cdot |||p|'||' = 0$$
(8) $$p' \cdot ||||p|'|||' = 0$$

P is not a solution of the equation $|p|' \cdot ||p|| = 0$ and consequently, by our principle does not satisfy

$$|p|' \cdot |||p||'|' = 0 \qquad |||p||'| \cdot ||p|| = 0$$

and, therefore, is not a solution of

(9) $$p|' \cdot ||||p||'||' = 0$$

(10) $$||||p||'|| \cdot |||p||'|' = 0$$

(11) $$|||p||'| \cdot ||||p||'|'| = 0$$

(12) $$||||p||'|'|' \cdot ||p|| = 0$$

P is not a solution of the equation $||p||' \cdot |||p||| = 0$ and, consequently, by our principle does not satisfy

(13) $$||p||' \cdot ||||p|||'|' = 0$$

(14) $$||||p|||'| \cdot ||p||| = 0$$

P is not a solution of the equation

(15) $$|||p|||' \cdot ||||p|||| = 0$$

From theorems (1)–(15) we have at once,

$P\|$	$\angle\ P\|'\|'\|\|$	is untrue
$P\|'\|'\|\|$	$\angle\ P\|'\|'\|$	is untrue
$P\|'\|'\|$	$\angle\ P\|'\|'\|'\|'$	is untrue
$P\|'\|'\|'\|'$	$\angle\ P\|'\|'$	is untrue
$P\|'\|'$	$\angle\ P\|'\|\|'\|$	is untrue
$P\|'\|\|'\|$	$\angle\ P\|'\|\|'$	is untrue
$P\|'\|\|'$	$\angle\ P\|'\|\|\|'$	is untrue
$P\|'\|\|\|'$	$\angle\ P$	is untrue
$P\|\|'\|\|'$	$\angle\ P\|$	is untrue
$P\|\|'\|'$	$\angle\ P\|\|'\|\|'$	is untrue
$P\|\|'\|'\|$	$\angle\ P\|\|'\|'$	is untrue
$P\|\|$	$\angle\ P\|\|'\|'\|$	is untrue
$P\|\|\|'\|'$	$\angle\ P\|\|$	is untrue
$P\|\|\|$	$\angle\ P\|\|\|'\|'$	is untrue
$P\|\|\|\|$	$\angle\ P\|\|\|$	is untrue

by means of the principle $(P\mathcal{Q} \angle o)' \angle (P \angle \mathcal{Q}')'$. The rest of the table is developed simply by changing P into P' in these results.

In order to simplify the general problem whose solution is given in the sequel it will not be necessary to state again the fundamental principle which expresses the modal function of a product of two variables as a sum of products of modal functions of the single variables. This formula given by,

$$|p\cdot q| = p\cdot q + p'\cdot q'|p|\cdot|q| + |p|\cdot|q'|' + |q|\cdot|p'|'$$

fails for the cases $pq = 0$, $p'q = 0$, $pq' = 0$. Let us verify it for the cases $q = p, p', 0, 1$. It will be necessary of course to have before us constantly the results of the table.

For the case of $q = p$ we have,

$$\begin{aligned} |p\cdot p| &= p\cdot p + p'\cdot|p| + |p|\cdot|p'|' + |p'|'\cdot|p| \\ &= p + p'\cdot|p| + |p'|' \\ &= p + p'\cdot|p| \\ &= |p| \end{aligned}$$

For the case of $q = p'$ we have,

$$|p\cdot p'| = p\cdot p' + p'\cdot p|p|\cdot|p'| + |p|\cdot|p|' + |p'|\cdot|p'|' = 0$$

For the cases $q = 0$, $q = 1$, we have

$$|p\cdot 0| = p\cdot 0 + p\cdot 1|p|\cdot|0| + |p|\cdot|1|' + |0|\cdot|p'|' = 0$$

$$|p\cdot 1| = p\cdot 1 + p\cdot 0|p|\cdot|1| + |p|\cdot|0|' + |1|\cdot|p'|' = |p|$$

Since the contradictory of a sum of products is the product of the parts contradicted separately, we have for the modal of a product contradicted,

$$(p' + q')\cdot(p + q + |p|' + |q|')\cdot(|p|' + |q'|)\cdot(|q|' + |p'|)$$

or, if we multiply the first two and the last two factors together,

$$(|p|' + |q|' + p'\cdot q + q'\cdot p)\cdot(|p|' + |q|' + |p'|\cdot|q'|)$$

and this reduces at once to,

$$|pq|' = |p|' + |q|' + p\cdot q'|p'| + qp'|q'|$$

The two formulas for the modal of a sum are already familiar as a part of the classical algebra. They are,

$$|p + q| = |p| + |q|$$
$$|p + q|' = |p|' \cdot |q|'$$

and our results are easily generalized for the case of modals of any order or of any degree. Thus, changing q into $q \cdot r$ and reducing,

$$|pqr| = pqr + p'q'r' \cdot |p| \cdot |q| \cdot |r|$$
$$+p'q' \cdot |p| \cdot |q| \cdot |r'|' + p'r' \cdot |p| \cdot |q'|' \cdot |r| + q'r' \cdot |p'|' \cdot |q| \cdot |r|$$
$$+|p| \cdot |q'|' \cdot |r'|' + |p'|' \cdot |q'|' \cdot |r| + |p'|' \cdot |q| \cdot |r'|'$$
$$|pqr|' = |p|' + |q|' + |r|'$$
$$+ p'q|q'| + p'r|r'| + q'p|p'| + q'r|r'| + r'p|p'| + r'q|q'|$$
$$|p + q + r| = |p| + |q| + |r|$$
$$|p + q + r|' = |p|' \cdot |q|' \cdot |r|'$$

The most general problem of logic is to construct all propositions whose truth is independent of any specific meaning which the terms may have. As far as my knowledge extends this statement of the problem was first made by E. A. Singer, Jr. about thirty years ago.

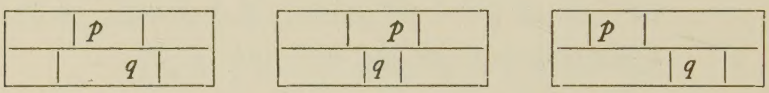
Consider now an arbitrary expression containing any number of variables, involving conjunctions, disjunctions and denials, together with any number of symbols of implication arranged in any way whatever. Such an expression, no matter what its form, may be represented as a modal function, and by repeated application of the formulas we have just given, may be finally reduced to a sum of products of modal functions of the single variables.

For this expression to be true it must reduce to unity, but since the denial of a sum of products reduces again to a sum of products by direct multiplication, the most general problem reduces to that of finding all the cases in which a sum of products of modal functions of the variables (ultimately the categorical forms) vanishes identically. But since for a sum to vanish each term

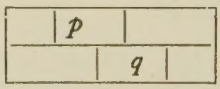
must vanish separately, the general problem may now be put in its simplest form:

To determine all the cases in which a product of modal functions of the categorical forms vanishes in independence of any meaning which the terms may have.

The expansion formulas fail when certain definite relations obtain between the variables p and q, but these happen to be the very cases for which no expansion formulas are needed. They are the cases in which pq, pq', $p'q$, one and only one, vanish. Let us represent them diagrammatically as follows:



the limiting cases $p = q$, q', 0, 1, for which the formulas hold, being excluded. Here $|pq|$ becomes respectively, $|p|$, $|q|$, 0. The formulas hold for the otherwise irreducible case:



as well as for all limiting cases.

In reducing any arbitrary expression, then, to a sum of modals of the categorical forms, we must consider this limitation at each step. That is, each time we employ the formulas we must keep in mind four possibilities, treating each recurrence of the modal operator as giving rise again to four possibilities, and so on, until we reach, as we must in a finite number of steps, the "atoms" of the system. It is of course a matter of no importance that the reduction may be complex. The important fact is that no obstacle can intervene to prevent our carrying it out. This possibility depends on the fact that we are always facing unambiguous alternatives.

The method of *quadrant analysis* which we shall now illustrate, enables us not only to determine the truth or untruth of any general statement involving logical relations and made about the elements, constant or variable, p, q, r, . . . , but furnishes us with

a general method of determining the consistency and independence of two or more propositional functions and their negatives. Such a general method has not been developed previously because up to now it has been impossible to handle the critical cases for which our expansion formulas hold. One instance from which the general method is easily abstracted, will serve to illustrate. Suppose:

$$P = (p \prec q')' + (q' \prec p)' \cdot \{(p \prec q) + (q \prec p)\}$$

$$\mathfrak{Q} = (p \prec q)' + (q \prec p)' \cdot \{(p \prec q') + (q' \prec p)\}$$

and we wish to know, among other things, if $P + \mathfrak{Q}$ is true, if P and $\mathfrak{Q}$ are consistent statements, if P implies $\mathfrak{Q}$, if $\mathfrak{Q}'$ implies P, and so on. We have:

$$P = |pq| + |p'q'| \cdot \{|pq'|' + |p'q|'\}$$

$$\mathfrak{Q} = |pq'| + |p'q| \cdot \{|pq|' + |p'q'|'\}$$

Case I. $pq = 0$, or $p \prec q', q \prec p'$.

$$P = |p'q'| \{|p|' + |q|'\}$$

$$= |p'| \cdot |q|' + |q'| \cdot |p|'$$

$$\mathfrak{Q} = |p| + |q|$$

$$P' = |p'|' + |q'|' + |p| \cdot |q|$$

$$\mathfrak{Q}' = |p|' \cdot |q|'$$

so that $P' \cdot \mathfrak{Q}' = 0$, $P + \mathfrak{Q}$ is true, $P' \prec \mathfrak{Q}$, $\mathfrak{Q}' \prec P$. These relations between P and $\mathfrak{Q}$ will turn out to be true in all cases.

Case II. $p'q = 0$, or $q \prec p, p' \prec q'$.

$$P = |q| + |p'| \qquad P' = |p'|' \cdot |q|'$$

$$\mathfrak{Q} = pq' + |p| \cdot |q|' + |q'| \cdot |p'|'$$

$$\mathfrak{Q}' = |p|' + |q'|' + p'|q| + q|p'|$$

and $P' \cdot \mathfrak{Q}' = 0$ as before.

Case III. $pq' = 0$, or $p \angle q, q' \angle p'$.

$$P = |p| + |q'| \qquad P' = |p|' \cdot |q'|'$$
$$\mathcal{Q} = |p'q| \{|p|' + |q'|'\}$$
$$= |p'| \cdot |q'|' + |q| \cdot |p|'$$
$$\mathcal{Q}' = |p'|' + |q|' + |p| \cdot |q'|$$

and $P' \cdot \mathcal{Q}' = 0$ as before.

Case IV. $p'q' = 0$, or $p' \angle q, q' \angle p$.

$$P = pq + |p| \cdot |q'|' + |q| \cdot |p'|'$$
$$P' = |p|' + |q|' + p'|q'| + q'|p'|$$
$$\mathcal{Q} = |p'| + |q'| \qquad \mathcal{Q}' = |p'|' \cdot |q'|'$$

and $P' \cdot \mathcal{Q}' = 0$ as before.

We have already verified the expansions when two or three of the four quadrants vanish. Consequently, for these cases, as for the case when no quadrant vanishes, we have after reduction:

$$P \cdot \mathcal{Q} = |pq| \cdot |pq'| + |pq'| \cdot |p'q'| \cdot |p'q|' + |p'q| \cdot |pq| \cdot |p'q'|'$$
$$= |p'|' \cdot |q| \cdot |q'| + |q'|' \cdot |p| \cdot |p'| + |q|' \cdot |p| \cdot |p'|$$
$$P' \cdot \mathcal{Q}' = |pq|' \cdot |p'q|' \cdot |pq'|' \cdot |p'q'|' |q|' \cdot |q'|' = 0$$

We can, accordingly, since the method is easily generalized for any number of variables, always reduce any functions $P, \mathcal{Q}, R \ldots$ involving the variables $p, q, r, \ldots$ to a sum of products of modals of the single variables. The subsequent chapters will develop all the cases in which a product of modals of the categorical forms implies zero. The most general problem of logic will then be resolved.

II

It is an opinion all but universally shared since Leibnitz' day, that the traditional logic is not true in all of its parts, if the terms are allowed to take on the meaning of empty classes. This criti-

cism repeated again and again, all turns on the same misunderstanding. It is assumed that the relation of "inclusion" as between classes can have only one meaning; that the "all, is" relation of Aristotle must be identified with, must have the same properties, as Boolean inclusion. The difficulty is overcome at once when we see that each of these relations is to be represented as a function, and not a very simple function, of the other. Let us express the "all a is b" of traditional logic, $a < b$, in terms of Boolean inclusion, $a \angle b$, as follows:

$$a < b = a \angle b \cdot \{b \angle a + (a \angle b')' \cdot (b' \angle a)'\}$$

A new form, "no a is b," is now gotten by changing the b above into b', and another "all non-a is b" by changing the a into a'. The remaining three categorical forms are the contradictories of these. We have stated their verbal interpretations on the first page of the last section.

From this definition all the characteristics of traditional logic follow at once without any restriction whatever being placed on the meaning of the terms. Thus at once,

$$a < a = a \angle a \cdot \{a \angle a + (a \angle a')' \cdot (a' \angle a)'\} = 1$$

$$a < a' = a \angle a' \cdot \{a' \angle a + (a \angle a)' \cdot (a \angle a)'\} = 0$$

so that we have the important results:

$$A(aa) = I(aa) = V(aa) = O(aa') = E(aa') = U(aa') = 1$$

$$A(aa') = I(aa') = V(aa') = O(aa) = E(aa) = U(aa) = 0$$

as soon as subalternation has been proved.

The truth of subalternation is of course the disputed point, but it follows at once from the definition:

$$\begin{aligned}(a < b) \cdot (a < b') &= (a \angle b) \cdot (a \angle b') \cdot (b \angle a) \cdot (b' \angle a) \\ &\angle (a \cdot a \angle b \cdot b') \cdot (b + b' \angle a + a) \\ &\angle (a \angle 0) \cdot (1 \angle a) \\ &\angle (1 \angle 0) \\ &\angle 0\end{aligned}$$

and it is easy to see that contraposition holds of A, U and O and that E, I and V are simply convertible. Finally, the transitive property of the "all, is" relation, that is the mood in *Barbara*, follows from the transitive character of Boolean inclusion. We have simply to express $A(ba)$, $A(cb)$ and $O(ca)$ in the manner of their definition and multiply the three expressions together. It will then be seen that each term in the product contains a zero factor and that consequently the sum of the terms vanishes identically.

We may equally well, if we start by assuming the Aristotelean system, derive the logic of Boole. This proof will be much simplified if certain extensions of Aristotle's system are established in advance. The theorems set down below and without proof follow at once from our definition.

1.1 $(a < b)\cdot(a' < a\cdot a')\cdot(b' < b\cdot b')' = 0$

1.2 $(a < b)\cdot(a < a\cdot a')'\cdot(b < b\cdot b') = 0$

2.1 $(a < b)\cdot(a\cdot b' < 0)' = 0$

2.2 $(a < a\cdot a')\cdot(a\cdot b' < 0)' = 0$

2.3? $(b' < b\cdot b')\cdot(a\cdot b' < 0)' = 0$

3.1 $(a < b)'\cdot(a\cdot b' < 0)\cdot(a < 0)'\cdot(b' < 0)' = 0$

4.1? $(a\cdot b < b)'\cdot(a\cdot b < 0)'\cdot(b' < 0)' = 0$

5.1 $(a < b)\cdot(c < d)\cdot(a\cdot c < b\cdot d)'\cdot(a\cdot c < 0)'.$
$\cdot(b' + d' < 0)' = 0$

5.2 $(c < d)\cdot(b' < 0)\cdot(b' + d' < 0)'\cdot(a\cdot c < 0)'.$
$\cdot(a\cdot c < b\cdot d)' = 0$

5.3 $(a < b)\cdot(d' < 0)\cdot(b' + d' < 0)'\cdot(a\cdot c < 0)'\cdot(a\cdot c < b\cdot d)' = 0$

5.4 $(b' < 0)\cdot(d' < 0)\cdot(b' + d' < 0)' = 0$

6.1 $(a\cdot b < c)\cdot(a\cdot c' < b')'\cdot(a\cdot c' < 0)'\cdot(b < 0)' = 0$

6.2 $(a\cdot b < 0)\cdot(b < 0)'\cdot(a\cdot c' < 0)'\cdot(a\cdot c' < b')' = 0$

7.1 $(a\cdot b < c)\cdot(d < b)\cdot(a\cdot d < c)'\cdot(a\cdot d < 0)'\cdot(c' < 0)' = 0$

7.2 $(a \cdot b < c) \cdot (b' < 0) \cdot (a \cdot d < c)' \cdot (a \cdot d < 0)' \cdot (c' < 0)' = 0$

7.3 $(a \cdot b < 0) \cdot (d < b) \cdot (a \cdot d < c)' \cdot (a \cdot d < 0)' \cdot (c' < 0)' = 0$

7.4 $(a \cdot b < 0) \cdot (b' < 0) \cdot (a \cdot d < 0)' \cdot (c' < 0)' \cdot (a \cdot d < c)' = 0$

The characteristic features of Boole's logic are now derived by means of the definition,

$$(a \angle b) = (a < b) + (a < a \cdot a') + (b' < b \cdot b')$$

We have simply to express each Boolean inclusion in each product below in terms of Aristotelean inclusion and multiply out the factors. The terms that do not obviously vanish will imply zero in virtue of the theorems noted under each head.

1. $(a \angle b) \cdot (b \angle c) \cdot (a \angle c)' = 0$

   by 1.1, 1.2.

2. $(a \angle b) \cdot (a \cdot b' \angle 0)' = 0$

   by 2.1, 2.2, 2.3.

3. $(a \angle b)' \cdot (a \cdot b' \angle 0) = 0$

   by 3.1.

4. $(a \cdot b \angle b)' = 0$

   by 4.1.

5. $(a \angle b) \cdot (c \angle d) \cdot (a \cdot c \angle b \cdot d)' = 0$

   by 5.1, 5.2, 5.3, 5.4.

6. $(a \cdot b \angle c) \cdot (a \cdot c' \angle b')' = 0$

   by 6.1, 6.2.

7. $(a \cdot b \angle c) \cdot (d \angle b) \cdot (a \cdot d \angle c)' = 0$

   by 7.1, 7.2, 7.3, 7.4.

Having deduced the logic of Aristotle from that of Boole and conversely, we become aware that we are confronted by two alternative but not irreconcilable systems. The contradiction which algebrists of logic take to be final, is a purely verbal one.

III

Having the general problem in mind as stated at the end of the first chapter, we proceed inductively toward its solution, beginning with the simplest case. If Mp and Nq stand for modal functions of p and q respectively, then immediate inference is the set of all propositions of the form,

(1) $Mp(ab) \angle Nq(ab)$

(2) $Mp(ab) \angle Nq(ba)$

wherein p and q are conceived as capable of taking on any one of the forms A, E, I, O, U, V. There will thus be thirty-six cases to consider under each head, these obtained by taking the permutations of the six letters two at a time. Set (1) and set (2) are called the array of immediate inference in the first and in the second figure respectively, difference of figure being conceived as a difference of term order.

Suppose that p has taken on the value A and q the value E, yielding a member of each set,

$$MA(ab) \angle NE(ab) \qquad MA(ab) \angle NE(ba)$$

and suppose that we substitute first $a = a'$, then $b = b'$, and finally $a = a', b = b'$. There would then result six new *moods* of the array,

$$MU(ab) \angle NA(ab) \qquad MU(ab) \angle NA(ba)$$

$$ME(ab) \angle NA(ab) \qquad ME(ab) \angle NA(ba)$$

$$MA(ab) \angle NU(ab) \qquad MA(ab) \angle NU(ba)$$

so that if we had once determined all the true and all the false variants of the first pair for all meanings of M and N, it will be a mere matter of bookkeeping to record the true and false cases of the other six. If instead of making $p = A$ and $q = E$, we had given them the values A,A and then A,O and then A,I and finally O,A I,A O,O O,I and had substituted the negative terms as before, all the other possible variants would have resulted. Therefore, in order to determine all the valid moods of immediate inference, it will only be necessary to consider these combinations.

Moreover, since the prime over any letter can be considered as part of the modal operator, and since these combinations can be written $A,A\ \ A,I'\ \ A,A'\ \ A,I\ \ A',A\ \ I,A\ \ A',A'\ \ A',I$, it will only be necessary to solve the arrays,

$$MA(ab) \angle NA(ab) \qquad MA(ab) \angle NA(ba)$$

$$MA(ab) \angle NI(ab) \qquad MI(ab) \angle NA(ab)$$

for since $I(ab)$ is simply convertible, that is, can be written $I(ba)$ the second figure in the last two instances is solved along with the first. The first of the four cases was solved in the first chapter, if we consider P to have everywhere the sense of A. In order to solve A,A in the second figure, it will be necessary to introduce a postulate that is connected with invalidity.

Existential Postulate: There exists an a, b, c, such that

$$O(ab) \angle o, \qquad O(cb) \angle o, \qquad I(ca) \angle o.$$

Theorem: There exists an a, b, such that

$$A(ab) \angle o, \qquad O(ba) \angle o.$$

The proof of this theorem involves principles that will be introduced later on. It is as follows:

$$|A(ba)| \cdot |A(cb)| \angle |I(ca)|$$

$$\therefore |A(ba)| \cdot |A'(cb)|' \angle |I(ca)|$$

$$\therefore |I(ca)|' \cdot |O(cb)|' \angle |A(ba)|'$$

But

$$|O(cb)|' = 1, \qquad |I(ca)|' = 1.$$

Therefore

$$|A(ba)|' = 1.$$

We are now in a position to show that there are no valid propositions in the array,

$$MA(ab) \angle NA(ba)$$

Suppose M and N are both affirmative modals and we substitute $a = b, b = a$, *the new* a, b *having the meaning of the theorem*, so as to get,

$$MA(ba) \angle NA(ab)$$

which is of the form $1 \angle 0$ and is invalid.

If M and N are both negative modals, we make $a = a, b = b$, in order to show that none of this type can be true.

Suppose M is affirmative and N negative and let $b = a$, so that we have,

$$MA(aa) \angle NA(aa)$$

If M is negative and N affirmative, we let $b = a'$, so that we have,

$$MA(aa') \angle NA(a'a)$$

and each one of these is invalid by the results of chapter II.

Suppose in the arrays $MA(ab) \angle NI(ab)$ and $MI(ab) \angle NA(ab)$ that M is affirmative and N negative. It will then appear if we identify a and b, that there are no valid moods in these sets.

Nor will there be any valid moods of these types for the case of M negative and N affirmative. In order to see this we have only to write $b = a'$. We now introduce a theorem connected with invalidity.

Theorem:

$$|A(ab)|'_n \cdot |E(ab)|'_n \angle 0 \text{ is untrue,}$$

for since the truth of $E(ab) \angle O(ba)$ has been established in the second chapter, and since $O(ba) \angle 0$, it follows that $E(ab) \angle 0$, when a and b have the meaning of the theorem. Therefore, if the above expression were true, so would the new

$$|A(ab)|'_n \cdot |E(ab)|'_n \angle 0$$

be true. But this is false. Hence the theorem. Let us now write two variants of this theorem:

(1) $$|A|'_n \cdot \angle \cdot |I'|_n \text{ is untrue}$$

(2) $$|I'|'_n \cdot \angle \cdot |A|_n \text{ is untrue}$$

Suppose that M and N are both negative. Then by means of (1) any mood of the array $MA(ab) \angle NI(ab)$ can be shown to be invalid by the principle,

$$(P \angle \mathfrak{Q}) \cdot (P \angle R)' \angle (\mathfrak{Q} \angle R)'$$
$$(P \angle R)' \cdot (\mathfrak{Q} \angle R) \angle (P \angle \mathfrak{Q})'$$

For example,

$$(A|||' \angle A'||'|') \cdot (A||| \angle I'|||)' \angle (A'||'|' \angle I'|||)'$$
$$(A||| \angle I'|||)' \cdot (I'|'|' \angle I'|||) \angle (A||| \angle I'|'|')'$$

Similarly, if M and N are both affirmative, then by means of (2) any mood of the array $MI(ab) \angle NA(ab)$ can be shown to be invalid by the same principle.

The remaining types are all valid. To show this we now introduce a fundamental induction formula:

Principle:

$$|A(ba)|_n \cdot |A(cb)|_n \angle |A(ca)|_n$$
$$\angle |A(ba)|_{n+1} \cdot |A(cb)|_{n+1} \angle |A(ca)|_{n+1}$$

Theorem:

$$|A(ba)|_n \cdot |A(cb)|_n \angle |A(ca)|_n$$

since in chapter II we have proved the antecedent for $n = 0$.

Theorem:

$$|A(ab)|_n \cdot |E(ab)|_n \angle 0$$

making $a = c'$ and since $A(cc') \angle 0$ by definition.

If two variants of this theorem are written down, viz.

$$|A(ab)|_n \angle |I'(ab)|'_n$$
$$|I'(ab)|_n \angle |A(ab)|'_n$$

all the remaining valid moods follow at once by the principle,

$$(P \angle \mathfrak{Q}) \cdot (\mathfrak{Q} \angle R) \angle (P \angle R)$$

For example

$$(A|| \angle I'||') \cdot (I'||' \angle I|||) \angle (A|| \angle I|||)$$
$$(A'||| \angle A|||) \cdot (A||| \angle I'|||') \angle (A'||| \angle I'|||')$$
$$(I'|| \angle A||') \cdot (A||' \angle A|'|) \angle (I'|| \angle A|'|)$$
$$(I||' \angle I'||) \cdot (I'|| \angle A||') \angle (I||' \angle A||')$$

IV

Proceeding inductively toward the solution of the general problem, our next type will be the syllogism. This will have one of the forms,

(1) $Lp(ba) \cdot Mq(cb) \angle Nr(ca)$

(2) $Lp(ab) \cdot Mq(cb) \angle Nr(ca)$

(3) $Lp(ba) \cdot Mq(bc) \angle Nr(ca)$

(4) $Lp(ab) \cdot Mq(bc) \angle Nr(ca)$

wherein as before p, q and r are conceived as capable of taking on any one of the values A, E, I, O, U, V. Thus we should have, if we take into account the permutations of the six letters three at a time, two-hundred and sixteen moods in each one of the four figures or term orders, which number will be multiplied again as we allow L, M, and N to vary.

Suppose now that p, q, and r have all the value A, and that we substitute in all four figures first $a = a'$, then $b = b'$, then $c = c'$. And beginning again, suppose we substitute first $a = a'$, $b = b'$, then $b = b'$, $c = c'$, and then $a = a'$, $c = c'$. Finally let us substitute $a = a'$, $b = b'$, $c = c'$. It is clear that a great many new moods will result and that once we have solved the array,

$$LA(a, b) \cdot MA(b, c) \angle NA(c, a)$$

it will be a mere matter of bookkeeping to record the true and false moods in the other equivalent instances. Now it is a fact that starting with the combinations *AAA*, *AAE*, *AAO*, *AAI*, *AOA*, *AOU*, *AOO*, *AOI*, *OOA*, *OOE*, *OOI*, *OOO*, and making these same substitutions, all of the two-hundred sixteen, or those

that remain, will result. Consequently, if we solve these cases we have all the others.

However, since these twelve combinations may be written, *AAA*, *AAI'*, *AAA'*, *AAI*, *AA'A*, *AA'V'*, *AA'A'*, *AA'I*, *A'A'A*, *A'A'I'*, *A'A'I*, *A'A'A'*, and since the prime of negation may be regarded as part of the modal operator, the complete solution of the syllogism is given with the solution of,

$$LA(a, b) \cdot MA(b, c) \angle NA(c, a)$$

$$LA(a, b) \cdot MA(b, c) \angle NI(c, a)$$

for $AA'V'$ turns into one of these on negating all the terms. We begin with the first case. Suppose:

(1) L affirmative, M affirmative, N negative.
Seen to be invalid on making $a = b$.

(2) L affirmative, M negative, N affirmative.
Seen to be invalid on making $a = b$.

(3) L negative, M affirmative, N affirmative.
Seen to be invalid on making $c = b$.

(4) L negative, M negative, N affirmative.
If we make $b = a'$ this reduces to

$$MI(ca) \angle NA(ca)$$

or

$$MV(ca) \angle NA(ca)$$

wherein M is affirmative and N is affirmative, and this is invalid.

(5) L negative, M negative, N negative.
This reduces to case (3) on applying the principle,

$$(P \cdot \mathfrak{Q} \angle R) \angle (P \cdot R' \angle \mathfrak{Q}')$$

We have, accordingly, only to consider the case in which L, M and N are all affirmative modals, for the other two cases will turn out to be variants of this one.

(6) L affirmative, M affirmative, N affirmative.

All the valid moods are in the first figure, for the second is seen to be invalid on making $c = b$, the third and fourth on making $a = b$. They are all a consequence of a generalized

$$|A(ba)|_n \cdot |A(cb)|_n \angle |A(ca)|_n$$

viz.

$$MA(ba)\cdot MA(cb) \angle MA(ca) \qquad [M \text{ affirmative}]$$

which follows from the
Principle:

$$[MA \angle NA]\,[MA(ba)MA(cb) \angle MA(ca)] \angle [NA(ba)NA(cb) \angle NA(ca)]$$

and what follows from this as a consequence of the principle,

$$(S \angle P)\cdot(P\cdot Q \angle R) \angle (S\cdot Q \angle R)$$

We might have employed in the beginning instead of this principle and the induction principle, the following:

$$(MAba\cdot MAcb \angle MAca) \angle (|MAba|\cdot|MAcb| \angle |MAca|)$$

$$(MAca \angle MAba + MAcb) \angle (|MAca| \angle |MAba| + |MAcb|)$$

Thus:

$$(Aba\cdot Acb \angle Aca) \angle (|Aba|\cdot|Acb| \angle |Aca|)$$

$$(|Aca|' \angle |Aba|' + |Acb|') \angle (||Aca|'| \angle ||Aba|'| + ||Acb|'|)$$

$$(||Aba|'|'\cdot||Acb|'|' \angle ||Aca|'|') \angle (|||Aba|'|'|\cdot|||Acb|'|'| \angle |||Aca|'|'|), \text{ etc.}$$

All moods not so derived can be shown to be invalid by identifying terms in one of the premises.

(7) L affirmative, M negative, N negative.

The valid moods are in the second figure and follow from those of the last case by the principle,

$$(P\cdot Q \angle R) \angle (P\cdot R' \angle Q')$$

And there are no more, for if there were such a mood it would be invalid under (6) by contradicting and interchanging M and N.

(8) L negative, M affirmative, N negative.

The valid moods are in the third figure and follow from those of (6) by a variant of the last principle,

$$(P\cdot Q \angle R) \angle (R'\cdot Q \angle P')$$

And there are no more for the same reason as given under (7).

The other type of syllogism, viz.

$$LA(a, b) \cdot MA(b, c) \angle NI(c, a)$$

may be treated as the last case except for the alternatives under (4), (6), (7), and (8).

(4) L negative, M negative, N affirmative.

If we make $c = a'$, this reduces to

$$MO(ab) \angle NE(ab)$$

or

$$MO(ab) \angle NU(ab)$$

wherein M and N new modals are both affirmative, and this is invalid.

(6) L affirmative, M affirmative, N affirmative.

The second figure is invalid by the existential postulate of the last chapter, but all the moods in the first, third and fourth figure are valid. We have,

$$|A(ca)|_n \cdot |E(ca)|_n \angle O \cdot \angle \cdot |A(ca)|_n \angle |I'(ca)|'_n$$

(a) $$|A(ba)|_n \cdot |A(cb)|_n \angle |I'(ca)|'_n$$

(b) $$|A(cb)|_n \cdot |A(ba)|_n \angle |I'(ac)|'_n$$

and the analogue in the third figure is derived as follows:

$$|A(ba')|_n \cdot |A(cb)|_n \angle |A(ca')|_n$$

$$\therefore \quad |E(ca)|'_n \cdot |A(cb)|_n \angle |E(ba)|'_n$$

$$|I'(ca)|'_n \cdot |A(cb)|_n \angle |I'(ba)|'_n$$

(c) $$|A(ca)|_n \cdot |A(cb)|_n \angle |I'(ba)|'_n$$

All the other variants follow from (a), (b) and (c) by

$$(S \angle P) \cdot (P \cdot Q \angle R) \angle (S \cdot Q \angle R)$$

$$(P \cdot Q \angle R) \cdot (R \angle S) \angle (P \cdot Q \angle S)$$

and cases (7) and (8) are treated as before.

V

A zero cycle is a product of modal functions of single categorical forms whose terms are arranged in cyclical order, the whole represented as implying zero. A product of cycles represented as implying zero, is called a zero conjunction of cycles. An intersection is a term which cycles or more generally chains of categorical forms have in common, and we say that two categorical forms are adjacent when they have a common term. A cross-product is any cycle formed at the intersections in a product of cycles or chains. A principle which we shall take for granted in its most general form is this:

If none of the cycles of a valid mood of the zero conjunction of cycles vanishes, then one of its cross-products vanishes.

Since the prime or symbol of denial over any letter can be regarded as part of the modal operator and since any unprimed letter can be changed into another primed letter, we shall habitually think of an affirmative form as an A, an I or a V modified by a modal operator containing an even number of primes, a negative form as an A, an I or a V modified by a modal operator containing an odd number of primes. The solution of the zero cycle is equivalent to the solution of the sorites.

The sorites is an implication of the general form,

$$X(1, 2) \cdot Y(2, 3) \ldots Z(n, n - 1) \angle W(n\ 1)$$

wherein $X, Y, \ldots Z, W$ are conceived to be governed by an arbitrary modal operator, expressed or understood, and wherein the comma between the terms indicates that the term order is unsettled. If we follow the convention of writing the major premise first, the term order in the conclusion is fixed as $(n\ 1)$.

Definition: If $X(ab) \angle Y(ab)$ but not conversely, then $X(ab)$ is said to be universal and $Y(ab)$ is said to be particular.

We proceed to enunciate certain theorems which will abbreviate the work of establishing the general solution that follows.

Theorem 1. If the conclusion is affirmative then all of the premises are affirmative.

Suppose that one or more premises are negative. Identify

terms (subject and predicate) in each one of the affirmative premises if there be any such, and suppress each one of these as if it were a unit multiplier. The resulting sorites containing only negative premises, can be reduced to an invalid mood of the syllogism or an invalid mood of immediate inference by identifying in succession terms that are next but one. The general method is easily abstracted from a single illustration. Suppose all the affirmative premises have been removed and that there remains,

$$E(12)\cdot U(23)\cdot O(34)\cdot E(45)\cdot U(56)\cdot O(67) \angle A(71)$$

Changing 3 into 1 and then 5 into 1, our sorites breaks up into a zero conjunction of cycles whose parts are,

$$E(12)\cdot U(21), \qquad O(14)\cdot E(41), \qquad U(16)\cdot O(67)\cdot O(71)$$

and which contain no other cross-products. Since none of these parts vanish separately, the original sorites is invalid.

Theorem 2. If the conclusion is negative then one and only one premise is negative.

For if all were affirmative the sorites would be reducible to an invalid syllogism by identifying in succession adjacent terms, and if more than one is negative the method under theorem 1 is applicable.

Theorem 3. If the conclusion is a universal modified by an affirmative modal operator, then all of the premises are universals modified by affirmative modal operators.

The only possibility we should have to consider is the presence of particular premises governed by affirmative modals, for a universal governed by a negative modal is the same as this, and a negative modal over a particular is the same as a universal modified by an affirmative modal. If the conclusion is A and one or more I- or V-premises occur we identify terms in succession being careful to preserve one of the I- or V-premises. If the conclusion is U or E one and only one premise is negative. If this premise is an O-form we identify terms in all the A-, I- and V-premises that occur. If an I- or V-premise occur and the negative premise is E or U, we identify terms in all the premises except the particular and the negative.

Theorem 4. If the conclusion is a particular modified by an affirmative modal operator, then not more than one premise is a particular modified by an affirmative modal operator.

Definition: If $Mp \angle Np$ but not conversely, the modal N is said to be weaker than M, and M is said to be stronger than N.

CONCLUSION IN THE *A*-FORM

If the conclusion is an *A*-form governed by an affirmative modal then all the premises are of the same form (theorems 1, 3) and it is easy to show that the modal of no premise is weaker than the modal of the conclusion.

In each premise the larger ordinal number must appear first (as subject). Otherwise, by the methods already employed, it will be possible to reduce the sorites to an invalid mood of the syllogism. The sorites has the form, then,

$$A(21) \cdot A(32) \ldots A(n\,n-1) \angle A(n\,1)$$

each premise being governed by an affirmative modal no weaker than the conclusion. It is evidently implied by the chain of valid syllogisms,

$$A(21) \cdot A(32) \angle A(31)$$
$$A(31) \cdot A(43) \angle A(41)$$
$$\ldots\ldots\ldots\ldots$$
$$A(n-1\,1) \cdot A(n\,n-1) \angle A(n\,1)$$

CONCLUSION IN THE *I*-FORM

The conclusion is affirmative, so that all the premises are affirmative (theorem 1), and particular, so that not more than one premise is particular (theorem 4), and it is easy to show that the particular cannot be in the *V*-form.

If all the premises are in the *A*-form, the first premise which presents its terms in the order $(s-1\,s)$ establishes that order in each premise which follows. For suppose a premise coming after the premise in question, should exhibit the term order $(s\,s-1)$. The sorites would then be reducible to an invalid syllogism of the form

$$A(s-2-1) \cdot A(s\,s-1) \angle I(s\,s-2)$$

Let us suppose that the term order $(s\ s-1)$ is preserved as far as the rth. premise and is then reversed. Then the order $(s-1\ s)$ is established from the rth. premise to the end and the sorites becomes,

$$A(21)\cdot A(32)\ldots A(r\ r-1)\cdot A(r\ r+1)\ldots A(n-1\ n)\ \angle\ I(n\ 1)$$

and this is constructible from the chain of valid syllogisms,

$$A(21)\cdot A(32)\ \angle\ A(31)$$

$$\ldots\ldots\ldots\ldots$$

$$A(r-1\ 1)\cdot A(r\ r-1)\ \angle\ A(r\ 1)$$

$$A(r\ 1)\cdot A(r\ r+1)\ \angle\ I(r+1\ 1)$$

$$I(r+1\ 1)\cdot A(r+1\ r+2)\ \angle\ I(I(r+2\ 1)$$

$$\ldots\ldots\ldots\ldots$$

$$I(n-1\ 1)\cdot A(n-1\ n)\ \angle\ I(n\ 1)$$

If the term order $(s\ s-1)$ is preserved from the first premise to the last, so that the form of the sorites is,

$$A(21)\cdot A(32)\ldots A(n\ n-1)\ \angle\ I\ (n\ 1)$$

then the chain of generating syllogisms will be,

$$A(21)\cdot A(32)\ \angle\ A(31)$$

$$A(31)\cdot A(43)\ \angle\ A(41)$$

$$\ldots\ldots\ldots\ldots$$

$$A(n-2\ 1)\cdot A(n-1\ n-2)\ \angle\ A(n-1\ 1)$$

$$A(n-1\ 1)\cdot A(n\ n-1)\ \angle\ I(n\ 1)$$

If the tth. premise is in the I-form, the term order in the first $t-1$ premises is established as $(s\ s-1)$. Otherwise, by the identification of terms, we could reduce the sorites to an invalid syllogism of the form,

$$A(s-1\ s)\cdot I(s\ s+1)\ \angle\ I(s+1\ s-1)$$

Similarly, the order of terms in the last $n-t-1$ premises is established as $(s-1\ s)$. For should any premise following the

I-premise present the term order $(s\ s-1)$, the mood of the sorites would reduce to an invalid syllogism of the form,

$$I(s-2\ s-1)\cdot A(s\ s-1) < I(s\ s-2)$$

Consequently the sorites has the form,

$$A(21)\ldots A(t\ t-1)\cdot I(t\ t+1)\cdot A(t+1\ t+2)\ldots A(n-1\ n) \angle I(n\ 1)$$

which may be built up from the chain of valid syllogisms,

$$A(21)\cdot A(32) \angle A(31)$$

$$\ldots\ldots\ldots\ldots$$

$$A(t-1\ 1)\cdot A(t\ t-1) \angle A(t\ 1)$$

$$A(t\ 1)\cdot I(t\ t+1) \angle I(t+1\ 1)$$

$$I(t+1\ 1)\cdot A(t+1\ t+2) \angle I(t+2\ 1)$$

$$\ldots\ldots\ldots\ldots$$

$$I(n-1\ 1)\cdot A(n-1\ n) \angle I(n\ 1)$$

the only restriction on the affirmative modal operators that govern the premises being that the affirmation of $I(t\ t+1)$ is no weaker than the affirmation of $I(n\ 1)$.

All the valid moods of the sorites with a V-conclusion can be gotten by negating all the terms in the sorites with an I-conclusion, and it is easy to show that there are no others. Those with an E-, U- or O-conclusion are obtained by contradicting and interchanging a premise and conclusion in one or other of the cases already determined.

ZERO CONJUNCTION OF CYCLES

In the solution of the sorites, which contains the solution of the zero cycle, we have already assumed an existential postulate: No zero conjunction of cycles can vanish unless one of the cycles or one of the cross products vanishes.

This principle evidently yields the solution of the zero conjunction of cycles, for if C be a cycle or cross-product in

$$C_1\cdot C_2\ldots C_n$$

then

$$(C \angle o) \angle (C_1 \cdot C_2 \ldots C_n \angle o)$$

CHAINS AND ZERO CONJUNCTIONS OF CHAINS

A chain or a conjunction of chains which are not cycles and which contain no cross-product, can evidently be reduced to unity by identifying terms in the affirmative forms and making them contradictory in the negative forms, and it is evident that a conjunction of chains can be reduced to a conjunction of the cycles it contains in the same way without introducing a zero factor.

The most general case we should have to consider would be a sum of products of chains represented as implying zero, since for a sum to vanish each term must vanish separately and because a product of sums can be reduced to a sum of products by direct multiplication. This case is now solved and with it the general problem we have proposed.

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An Examination of the Quantum Theories III

BY

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A CHANGE OF HEART

THE Bohr formulation—the original one—is now largely a matter of history. The usual career of a theory was its fate. It explained much, but it failed at the first signs of complication, even for the simplest molecules as hydrogen and helium. Some results were either only approximate in a loose or incomplete fashion, e.g. in the application of the correspondence principle to intensities, the only reliable predictions being only for an absence of certain lines, or they quite disagreed with experiment, e.g. in the case of the hydrogen molecule and the hydrogen molecule ion, the normal and excited states of helium, the anomalous Zeeman effect,[1] the Ramsauer effect (the phenomenon of the deflection of electrons at low speeds being less than at higher speeds in atomic collisions), and in the treatment of problems where more than one electron was involved. In the last mentioned cases the perturbation method, so successful in solving "many-body" problems in celestial mechanics, failed completely because systems of several electrons, possessing frequencies of the order of light waves, cannot interact classically. A bit of much forced pragmatism was practiced in treating the

[1] In this case a semi-empirical solution was obtained by the use of 4 quantum numbers (incorporating the idea of electron spin) but *not* by solving a Hamilton-Jacobi partial differential equation.

many electron atom as a variation of the hydrogen atom. Besides, there were technical oddities, like $\frac{1}{2}$ quantum numbers, and unpenetrated anologies between visible and X-ray spectra.

Failure in the very midst of success was particularly bewildering in this case. Overshadowing all was the sense of Planck's original sin. Yet there was no going back to the old conservatisms. There were to be new radicalisms for old, genuine ones or ones with a vengeance. A first measure of despair was taken by Bohr, Kramers and Slater in permitting the electron to radiate *while* in its orbit; at any rate to emit at least some energy before passing into the next. This was proposed in order to remove a mystery that seemed to reside in the question of radiation *during* transition. If the frequency of radiation depended on the frequency in the orbit of destination as well as on the frequency of the starting orbit, as the fundamental postulate requires, to put it bluntly, how could the electron know where it was to go? This way of sidestepping a kind of prevision on the part of the electron introduced, however, the startling alternative that for atomic systems the principle of conservation of energy is only a statistical one, i.e. need not hold for each individual electron.

THE COMPTON EFFECT

Counter to this went the explanation of the recently discovered Compton Effect, which allowed the retention of a completely deterministic conservation principle. Compton found that when an X-ray collides with a loosely bound electron, a secondary X-ray bounces off which is of lower frequency, and that the collision is of the nature of billiard ball collisions, the electron flying off with an increased momentum in some other direction. In each collision energy and momentum are conserved. One can draw the well-known triangle of forces, if one considers the X-rays as a hail of photons or energy quanta or, in other words, of "particles" of energy. Although the conservation principles are not violated the recoil of the electron and the photon can take place in an infinite number of ways and still balance out. Perhaps a statistical principle resides in this recoil rather than in the conservation principle?

THE CALL TO "OBSERVABLES"

Next, Kramers and Heisenberg, working on the dispersion of light, hit on the method of expressing results in terms of the frequencies and intensities of the very spectral lines they were observing. This developed into Heisenberg's ambitious project of formulating laws only in terms of "observables." There was to be no more talk of orbits, orbital phases and frequencies, and the like, which are apparently unobservable into all eternity. The mathematics used, however, could be anything whatever, provided it gave final results in terms of observables, and as a matter of fact complex numbers devoid of any physical meaning were the first choice. Perhaps to satisfy old habits one might speak of "virtual oscillators."

Maxims (often referred to as principles) began to appear in the literature. "The true laws of nature are relations between magnitudes which must be fundamentally observable." (Born). It was recognized more sharply than ever that a great part of the appeal in the relativity theory lay in obedience to this maxim, as well as in its ambition to state the laws of nature in invariant forms (independently of the system of coördinates). Unobservables are the position, velocity, period of the electron; observables are energy levels, light frequencies, intensities, polarizations, and over all dimensions of the atom (via the kinetic theory of gases). The notion of an "instantaneous state," which is nothing but the definition of phase at a given instant, was made to vanish along with the conception of an orbit. Transition from instantaneous state to instantaneous state hence also lost meaning.

MATRIX MECHANICS

The Matrix mechanics (1925) of Heisenberg was a development in the above spirit. He was compelled to deal, not with single numbers but with relations between *sets* of numbers (matrices) for which an unusual kind of algebra holds, a non-commutative one. There were also "matrix quantum conditions" to fix physical meaning, (the form $qp - pq = \frac{hi}{2\pi}$, where q

and p are matrices) to derive energy levels, and fix frequencies. Another way of putting it is that Heisenberg replaced the Fourier developments of a classical mechanical quantity by a matrix scheme, the elements of which symbolize purely harmonic vibrations and are associated with the possible transitions between the old "stationary states," said to be "a calculus with directly observable quantities." The frequencies (empirically) still obeyed the old combination principle and the correspondence principle was retained. The method proved powerful since it solved the cases of the more complex atoms which floored the old theory. This has been described as a change from mechanism to pure formalism—and so it is, if we think only of the mathematical character of the matrices. The tendency, however, is to forget that in practice quantum conditions are postulated in addition (to fix the frequency of radiation) and that such a procedure at least re-admits something of the Bohr model.

What is definitely and neatly discarded of the older models by this formalism is the notion of an oscillating (electrical) doublet, a notion which threatened two difficulties, namely (1) the spectrum of an atom would be invariable or independent of excitation and (2) the radiation would never stop or some magical action without reaction would be required. This in turn sets a problem of enumerating (statistically) the number of atoms in the various energy states.

THE WAVE EQUATION

History deals in coincidences. Another point of view was advanced by Schroedinger almost at the same time. Inspired by DeBroglie's attempt to complete the analogy between optics and dynamics as originally suggested by Hamilton (parallelism between the principles of Fermat and Hamilton, i.e. the principles of minimum optical path and of least action) he proposed an equation for the propagation of what may be called *matter-waves*, the Ψ function, couched in the mathematics suitable for an n-dimensional oscillator system. The equation is a differential equation. This, operating in a mathematical continuum, involves the traditional notion of causality—but with a difference.

In 1927 Jordan called it "statistical causality," since the differential equation does not apply, as in ordinary mechanics to the motion of individual corpuscles, but to a probability function which governs the occurrence of certain quantum phenomena. Dual aspects are involved, wave and corpuscle, the function being the most general form for the expression of wave-motion associated with any corpuscle or system of corpuscles in motion. On account of the probability aspect in the Schroedinger interpretation, transition between two states can be said (somewhat figuratively) to signify the *coexistence* of the two states. The measure of probability of transition was shown to be the square of the function, Ψ^2. This may also be said to give the *density* of points in a space. The conception of a definite orbit is replaced by that of a certain region in space (surrounding the nucleus) for which the probability of occurrence of the electron is high. The waves are waves of probability. The electron is a "smeared-out" electron. Of course, there are really no stationary states. Instead there is something (Schroedinger, 1927) which is to be described as a "resonance phenomenon" rather than the discontinuous exchange of energy between atoms required by the directly-spoken quantum postulate of Bohr.

To put the idea of wave mechanics more popularly one may say at this point that it is a grand elaboration of the violin string analogy, but it is nothing so grossly concrete, dealing only with what Sommerfeld called "die Schwingung eines abstrakten Etwas," the oscillation of an abstract something. Another suggestive analogy is to look upon wave mechanics as having the same relation to ordinary (macro) mechanics as wave optics has to ray optics. And it must be repeated one must not forget that at the same time wave mechanics is a statistical mechanics of swarms and "Ladungswolke" (cloud of charge) depending on the definiteness of the position of the electron, whose charge is continuously distributed. In this connection it is also not quite fair to say, as is often done, that the Bohr model was discarded. It was made use of in an n-dimensional way.[2] Another misconception is that

[2] The technical argument cannot be given here. Schroedinger's stating of the total energy in Hamiltonian form with the potential energy entering involves *implicitly* an *n*-atomic *model.*

continuity is completely abandoned. The fluctuation or velocity of the electrical density or "waves of probability" obeys the equation of continuity. Thus, and only thus, arose the mode of speech of "a wave motion associated with a particle," that is, merely where one is the other also is, neither being the productive cause of the other. These waves, matter-waves, are not electromagnetic waves. Mathematically they are not even real quantities and so when they are said to be "diffracted" by crystals, it is done rather figuratively.

TRANSFORMATION THEORY

A demonstration was early available that the matrix method and the wave statistical method were but two forms of a more general mathematical approach. Therein was mutual reinforcement and a simplification of the solution of certain problems. A little later there appeared the more powerful transformation theory, which completed by statistical theory, included the previous forms of quantum mechanics as special cases (Born, Dirac, Jordan, Pauli, 1927). The means employed, such as the q numbers of Dirac or the operators changing one set of variables to another, are more general than matrices. In a space of an infinite number of dimensions with each point a function, the operators change one point to another. All the operators are linear, and one works with the wave function Ψ, rather than a distribution function. All this in obedience to the central problem of mechanics: given the initial ensemble, to find the final. Promptly there were no difficulties wanting, let us say for the quantum theory, version of 1929: (1) there was still a puzzlement concerning the meaning of the Pauli principle, (2) Dirac produced four wave functions to cover both electron spin and relativity, but only two were needed,—and there was a finite chance for a change of sign of the electron, (3) spontaneous radiation, and the ratio of electron to proton mass were a challenge, (4) the nucleus was thoroughly problematic as to its structure and (5) the choice of coordinates in the wave equation—the so-called symmetrization difficulty—was puzzling. But by such signs one was to conquer.

EXPERIMENTS, CORPUSCLES AND WAVES

Nearly at the same time experimental demonstrations caught up with surprising promptness with the theoretical efforts. Davisson and Germer showed that electrons of moderate velocities (20–200v.) showed formally analogous behavior in diffraction from single crystals (of nickel) in the manner of waves, e.g. X-rays. G. P. Thomson did the same kind of thing for high velocity electrons (15,000–30,000 v.) through thin metal foils. These *corpuscles* behaved exactly like *waves*, following a precise relation laid down by DeBroglie.

The vital problem of quantum mechanics became: How can one reconcile the seemingly contradictory characters of particles and waves? Was there not some principle back of the persistence with which particles associated with themselves wave motions? and vice versa?

Davisson took the very practical attitude of the working scientist in the declaration: "I believe, however, that for the present and for a long time to come we shall, in describing experiments worry but little about ultimate realities and logical consistency. We will describe each phenomenon in whatever terms we find most convenient."

Compton had shown that *light* somehow is both wave motion and a stream of particles. Davisson and Germer had shown that a *beam of electrons* is both a stream of particles and a wave motion. Davisson had proceeded in the usual manner, conceiving electron scattering in a classical fashion, namely the interaction of flying particles with a central field, perhaps attending to the prediction of W. Elsasser (Naturwissenschaften 13, 711 of 1925) that evidence for wave-mechanics would be found in the interaction of a beam of electrons and a crystal, but surely not from a logical analysis intended to resolve contradictory notions. Davisson's attitude was: Here are the experiments, and its up to the logic to explain them, and if the logic can't, go on just the same.

THE PRINCIPLE OF INDETERMINACY

The critical heroic step at this stage was taken by Heisenberg in stating a principle which attempts to make natural the "asso-

ciation" of wave with particle. The particle (or corpuscle) is defined by its momentum, p, and by its instantaneous position, q. The particle possesses energy which presumably travels with the particle. If the course of a beam of light is an infinite monochromatic plane wave, it becomes impossible to say just where the energy (or the particle) is. If we have a group of waves, these add to a resultant wave with a crest within which the particle is located, and with which it moves. To locate the particle sharply it is necessary to get a sharp crest, but this unfortunately means a greater range of wave-lengths. To get the momentum or velocity, however, with sharpness, it is necessary to narrow the range of wave lengths. The increased sharpness of the one is obtained at the cost of a decreased sharpness of the other. Nor is one speaking here of what is ordinarily called precision of measurement. We are so-to-speak at the pre-apparatus level. The principle states "When dealing with amounts of action comparable with Planck's h, one cannot assign accurate numerical values simultaneously to a coordinate and its corresponding momentum." It actually proposes to legislate what experiments are impossible, because of a fundamental ambiguity. It also follows, that if we can never get *initial* conditions without some ambiguity, the *final* conditions will carry the taint. All we can hope is to derive the *probability* of a future state, a matter of so-called *statistical determination*. J. H. Van Vleck (Chemical Reviews V, 470 (1928)) gives a table which is reproduced here:

CLASSICAL DETERMINISM	QUANTUM INDETERMINISM
Given initially x, y, z, v_x, v_y, v_z	Given v_x, v_y, v_z accurately; then x, y, z, are undetermined (i.e., all values of x, y, z, are equally probable), or vice versa x, y, z, given and v_x, v_y, v_z undetermined.
↓ (Arrow indicates calculation by classical dynamics)	↓ (Arrow indicates calculation by quantum dynamics)
Accurate determination of x, y, z, v_x, v_y, v_z at all subsequent times.	Only statistical determination of x, y, z, v_x, v_y, v_z at subsequent times.

The principle of indetermination, or indeterminacy, or uncertainty, or indefiniteness, or "Unbestimmtheit," has all the engaging quality of a philosophic dogma, and at the same time the

requisite mathematico-physical precision. The form $\Delta p . \Delta q \sim h$ is the one most frequently seen, which says that the product of the error of specifying momentum p and of the error of specifying the coördinate q is of the order of magnitude of Planck's constant h. As you decrease the one you increase the other. This resembles greatly the original Planck sin of setting $E/\nu = h$. In fact there is another form of the principle which limits the accuracy of simultaneous determination of time and energy $\Delta E . \Delta t \sim h$, which obviously is the old wolf in sheep's clothing.

This, of course, became a challenge (1) to derive other forms of the principle in numerous situations in order to demonstrate its universal significance in the microscopic realm where the errors will be of the order of h, and (2) to derive the paraphernalia of wave mechanics as *consequences* of the principle. Numerous discussions of ideal or "thought-experiments" (Gedankenexperimente) where the *interaction* of the thing measured and the measuring instrument was analyzed, led to a series of pairs of indefinites which obey the multiplication rule of Heisenberg's principle. Bohr saw in it a universal character, harking back to the distinction between subject and object, and described the relationship by a broader term than indeterminacy, namely "complementarity." His philosophic analysis tried to dispose of the wave-particle contradiction by the insistence that experiments which evoke the wave character and those that evoke the particle character are mutually exclusive,—but of these things we shall speak later. The second main objective is illustrable by such contributions as of Lindemann (1932) where an attempt is made to show that the exclusion principle and quantization of energy are consequences of Heisenberg's principle. There's reason to doubt the success of such efforts, but if they were beyond question, i.e. if the wave equation, etc. were fully derived, the principle of indeterminacy would be really a mighty postulate indeed.

In order to complete our story of the scientific aspects we ought to point out that healthy physicists had more straightforward chores to do with the indeterminacy principle and wave mechanics, than to deny all validity to ideas of continuity, causality, or to set up defenses for free-will on the human level. Besides

the internal task of setting up a structure of wave mechanics on the basis of the indeterminacy principle and other problems we mentioned above, there was the external task of reconciling the results of quantum theory with relativity. A recent conquest of the sort is Born's combination of modified Maxwell equations for the electromagnetic field with quantum theory. The old field equations did not account for the existence of a "radius" of an electron, that is, its finite energy or mass. The new ones do with a quantization of the electromagnetic field in time-space. The discovery of the positron, the neutron and the new forms of hydrogen, has unloosed many new problems as well. And, finally, side by side with the indeterminacy principle, which by the postulated ambiguity of initial conditions leads to the necessity of stating final states only as probabilities, there has also been a determined development of statistics, foreshadowing the emergence of statistical postulates.

Our further discussion will deal more specifically with the logic of the situation as it has developed up-to-date, and how it affects philosophical implications.

SUMMARIZING STATEMENT

1. The "Quantum Renaissance" of the last decade is described.
2. An attempt to scuttle the conservation principles had its defeat in the Compton effect.
3. Quantum Mechanics was called to attend to "observables" in the fashion of Relativity.
4. The "pure formalisms" of matrices, wave-equations, etc. were made to yield physical observables by added postulates.
5. It is still a question whether the Bohr model has been *completely* discarded.
6. Both energy and matter have been made to reveal both a particle and a wave character, which becomes very significant on the atomic microscopic level.
7. A very general principle of indeterminacy or complementarity sets limitations to observation in principle, and reveals the wave and particle aspects as limiting pure cases of the actual.

8. The welding of relativity with quantum theory, in so far as it has been accomplished, raises hopes of a clearer logical synthesis of points of view which still sound contradictory.

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Physico-Mathematical Aspects of the Gestalt-Problem

BY

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FOLLOWING the general program, outlined in a previous paper on the Foundations of Mathematical Biophysics,[1] we shall outline some physico-mathematical aspects of the Gestalt-problem. Faithful to the principles laid down before, we shall not attempt to give a theory of such and such psycho-physiological phenomena connected with our problem, but we shall investigate in a preliminary way the *abstract conceptual basis*, which *must* underlie the construction of any concrete theory. To make our aim quite clear and to avoid at the same time unnecessary repetition, we refer the reader to our previous paper.[1] Furthermore we shall give here only an outline of the physico-mathematical aspects; and of necessity we must leave a number of details untouched and unanswered.

We shall consider systems consisting of a very great number of elements possessing the following properties:

(1) Some of the elements, which we shall call *a*-elements, react by some kind of physico-chemical change to the changes of parameters which characterize the environment (temperature, pressure, light, electric charges, etc.). This kind of change of environmental parameters we shall call for brevity "stimulus," and the reaction of an *a*-element its "excitation."

(2) Some of the elements, *e*-elements, react to the excitation of

[1] Philosophy of Science, *1*, 176, 1934.

some of the *a*-elements. The excitation of the latter thus serve as stimuli for the former.

(3) Each *e*-element has at least one *a*-element (*a'*-element) which can stimulate it. But some *a*-elements do not stimulate any *e*-elements. Stimulation of such an *a*-element (a_1-element) does not produce any *e*-reactions. But if an a_1-element be excited simultaneously with an *a'*-element, then this a_1-element becomes itself an *a'*-element, and its stimulation will elicit a reaction of the *e*-element which was previously elicited by stimulation of *a'* only. We say that the reaction, *e*, has become conditioned to the stimulus, a_1.

(4) When after conditioning a_1 to *e*, the a_1-stimulus is repeated several times without being followed by the *a'*-stimulus, the a_1-stimulus looses its property of eliciting the *e*-reaction. We shall call this "inhibition."

(5) The stimulation of an *a*-element does not limit the excitation to this particular element. The excitation spreads to other elements according to some law, which we do not define here any closer. The intensity of excitation of any *a*-element is therefore a function of the intensities of excitation of all other *a*-elements.

In some regions of the system the excitation of an element may inhibit the excitation of the others.

(6) In order that an excitation of one element should produce an excitation of an adjacent element, it is necessary that the intensity of excitation of the former exceed a certain value, called the threshold of the latter.

(7) The *a* and *e*-elements are not necessary intermingled. In general they may be separated and lie in different regions of the system. The excitation of an *a*-element by an external stimulus causes an excitation of a number of intermediate elements, *i*-elements, which finally pass the excitation on to an *e*-element.

For our present discussion we do not need to consider any closer the physico-chemical nature of the reactions involved or of the mechanism of conditioning. Various suggestions in this direction have been made in our previous publications.[2] But the fol-

[2] Zt. f. Phys. *58*, 523, 1929; Jour. Gen. Psych. *5*, 207 and 368, 1931.

lowing considerations will hold regardless of any such special assumptions.

When speaking of the intensity of excitation, we refer to the intensity of the physico-chemical factors involved in the process, whatever their nature may be. These may be an electric potential, or the concentration of some substance or the permeability, surface tension, etc. In a given region of the system the intensity of excitation will be proportional to the product of the density of elements and the intensity of excitation of each element.

Let us now consider not single stimuli, but any spatio-temporal stimulus patterns. For instance, elements a_1, a_3, $a_7 \ldots a_i$, a_k, a_p, may be all stimulated simultaneously forming a spatial stimulus pattern. Or the elements may be stimulated in a definite succession, thus a_7, a_1, $a_{12} \ldots a_p, \ldots a_k$, etc. forming a temporal stimulus pattern. Or a combination of both may occur.

Any spatio-temporal pattern produces in the system a definite distribution of excitation, E, which varies in general with time. But no matter how complicated the distribution of E is, it is the same for a given stimulus pattern.

The i-elements of a region do not all have the same threshold. Their thresholds are distributed according to the law of chance. We have

$$N(h) = NU(h); \int_0^\infty U(h)dh = 1 \qquad (1)$$

where N is the total number of elements, and $N(h) = N\,U(h)$ is the number of elements with the threshold, h. Only those elements of the region (x, y, z) are excited, whose thresholds, h, are smaller than $E(x, y, z)$, and this number is equal to

$$N(x, y, z) = \int_0^{E(x,y,z)} N(h)dh \qquad (2)$$

$N(x, y, z)$ is therefore a function of the coördinates, which itself is determined by another function of coördinates, namely $E(x, y, z)$.

Since $E(x, y, z)$ is characteristic of a stimulus pattern, so will

$N(x, y, z)$ be. The total number of elements, which are excited, is given by the integral

$$\mathfrak{N} = \int N(x, y, z)\, dxdydz \qquad (3)$$

taken over the whole volume of the system, involved in the excitation spread, and is also a characteristic of the stimulus pattern.

If an unconditioned stimulus is applied simultaneously with $E(x, y, z)$ so as to produce an unconditioned reaction pattern, which involves a number $\overline{N}$ of e-elements, then *only* the number of a-elements, given by (3), will become conditioned to those e-elements.

It may of course happen that two different patterns will produce the same $\mathfrak{N}$. But, unless the patterns are identical, though the total number of conditioned elements may be same, the individual elements involved will be different

We may characterize the stimulus pattern by the function $E(x, y, z)$ and speak thus of the "pattern $E(x, y, z)$."

Let us condition the pattern $E(x, y, z)$ to a given reaction R. And consider now a different pattern $E_1(x, y, z)$.

In conditioning $E(x, y, z)$ $\mathfrak{N}$ elements are involved; in conditioning $E_1(x, y, z)$, $\mathfrak{N}_1$ elements are involved. There will be a certain number of elements $\mathfrak{N}^*$ common to both cases. Therefore if $E(x, y, z)$ is conditioned to R, and $E_1(x, y, z)$ is then produced, there will be a response R through the $\mathfrak{N}^*$ common elements. If however $E_1(x, y, z)$ is repeated without reinforcement, so as to become internally inhibited, then this inhibition will affect also the $\mathfrak{N}^*$ common elements. If however $E(x, y, z)$ is produced, it will still produce R, through the $\mathfrak{N} - \mathfrak{N}^*$ elements which were not common to $E(x, y, z)$ and $E_1(x, y, z)$. Thus we can produce a response to a specific pattern by differential inhibition, because the procedure may be applied to any number of patterns E, E_1, E_2, . . . etc.

Remembering that $E(x, y, z)$ varies in general with time, and must really be written

$$E(x, y, z, t)$$

we find that different elements will be involved in the conditioning of the same stimulus pattern, if the unconditioned stimulus is

applied at different time-intervals after the conditioned one (delayed reflex).

The variation of $E(x, y, z)$ with time leads also to the following consideration:

The excitation E_o of an afferent center, C, at the time $t = O$ produces at any time $t > O$ an excitation pattern $E(x, y, z, t)$. But the process of spreading of the excitation is irreversible (cf. the irreciprocity of the nervous centers). Therefore if we produce in some other way a pattern $E(x, y, z)$, which at a given moment is identical with $E(x, y, z, t)$, this will not result, in general in the production of E_o at C.

If the conditioned stimulus is applied *before the unconditioned*, then the application of the unconditioned stimulus E_u^o results in a connection of the elements producing it with those involved in the pattern $E_c(x, y, z, t)$. And every time that $E_c(x, y, z, t)$ is produced, E_u^o will also be produced. But E_u^o produces directly the response R. If however the unconditioned stimulus is produced first, then the conditioned stimulus E_c^o becomes associated with the pattern $E_u(x, y, z, t)$, produced by the unconditioned stimulus. Every time E_c^o is produced, it will produce also $E_u(x, y, z, t)$; but this will *not* produce, as we have seen, E_u^o, and hence will not produce R. This reminds one of the general failure to produce reversed conditioned reflexes.

We now come to the study of one of the most important and difficult aspects of the Gestalt-problem. We have seen how differentiated responses to stimulus-patterns may be produced. But when we consider a case of an animal conditioned to respond to a square, no matter what position the square occupies and how large it is, but not to triangles or circles, we see that we have here to deal with a much more complicated phenomenon. When a square is presented to us in various positions, we still recognize it as a square, in other words we respond to it always in the same way, although the retinal elements involved are each time quite different and therefore the corresponding $E(x, y, z)$ varies also from case to case. Again the square may be of any color, or of the same color and of varying brightness. It may be drawn in black lines on a white background, or in white lines on a black background. It is still recognized as a square.

We must investigate therefore how the same stimulus-patterns, which however involve *different* elements, may finally produce the same response.

As we have seen, the action of one element on another varies from one region of the system to another. Consider a system, in which the inhibition is very strong. In that case stimulation of a number of adjacent elements, which fill densely an area limited by a closed contour, Γ, on a surface, S, will not result in any appreciable excitation, except for those elements, which lie on the contour. For each element inside the coutour is surrounded by a number of inhibiting elements, whose action is very strong. Each element on the contour has however only half as many inhibiting adjacent elements and will therefore be excited. If instead of stimulating all elements *inside* the contour, we stimulate all elements *outside* of it, the effect will be the same: only elements along the contour will be appreciably excited. The excitation of each of those elements will depend on the intensity, I, of the stimulus at that point as well as on the intensity of excitation of all other elements on the contour, and will be therefore represented by an integral equation

$$E(s) = I(s) + \alpha \int E(s')\, K(s, s')\, ds' \qquad (4)$$

ds and ds' denoting elements of the contour line. $K(s, s')$ is determined by the law of interaction of the elements, and by the *shape* of the contour curve.

The excitation at any point of the curve will therefore be determined by the geometrical shape of the whole curve, Γ.

Let us now consider one particular structure of our system. Let the elements with various thresholds, h, be distributed at random on a surface, S, so that every region of S contains on the average the same number of elements of threshold h, as any other similar region. But let *all* elements of the same threshold h_i conduct away their excitation to the same region A_i in some other part of the system. All elements with a threshold h_k conduct away their excitation to a region A_k, which may be adjacent, but different from A_i and so on. The totality of regions A_i we shall call region A. If $K(s, s')$ is a function of the distance r between

s and s' only, that is if the region S is homogenous and isotropic, then it follows that no matter how we displace the contour on S, the excitation pattern in A will remain the same, as long as the *size* and *shape* of the contour remain constant.

If the excitation pattern in A is conditioned to a response R, then a geometrical form will cause the response R as long as it preserves its *shape* and *size*, but regardless of its position on S, as well as regardless whether the stimulus-pattern is limited by the contour Γ from outside or from inside. Although different elements will be involved each time, the number of elements of a given threshold h_i will always remain the same. For these elements are scattered at random on S, and the homogeneity of S makes (4) invariant with respect to any displacement of Γ as a whole.

Hence, if $K(s, s') = K(r)$, the excitation pattern at A is invariant with respect to any displacements of Γ on S.

Let us see, what other conditions must be satisfied by $K(r)$, in order that an invariance may be obtained also with respect to a simple change in size of Γ, that is for the transformation:

$$x_1 = \lambda x; \qquad y_1 = \lambda y; \qquad z_1 = \lambda z \tag{5}$$

The increase in size λ times of the contour on S will involve λ times more elements. But the region ds will also have increased λ times, so that the number of elements per unit length, which determines $I(s)$ will remain the same. Hence the only requirement for the invariance of (4) with respect to (5) is the invariance of $K(s, s')ds' = K(r)ds'$. But this requires that

$$K(r) = \frac{A}{r} \tag{6}$$

Under these conditions the values of $E(s)$ in the corresponding point of Γ and $\lambda\Gamma$ will be the same. Hence at the corresponding points elements only up to the same threshold will be excited. But because of $ds_1 = \lambda ds$ the number of those elements will have increased λ times. The result of this is, that although the relative intensities of excitation of the various regions in A will remain the same, their absolute value will have increased λ times. If

the differential equations, which govern the propagation of excitation, are linear, then we shall have in A

$$E(A)_1(x, y, z) = \lambda E(A)(x, y, z) \tag{7}$$

Thus the change in size of Γ leaves the form of the excitation pattern in A the same and only multiplies all values with λ.

However E and λE must be considered as different patterns and the same response will still not be produced by Γ and $\lambda\Gamma$. If we wish to obtain entirely identical patterns from Γ and Γ' we must consider a more complicated structure of our system, and assume that between A and the conditioning center an additional center is interposed, which reduces E and λE to exactly the same pattern. The structure and properties of these centers may be very similar to that of S. In fact we have seen that the elements in S will not become excited for an uniform distribution of I,

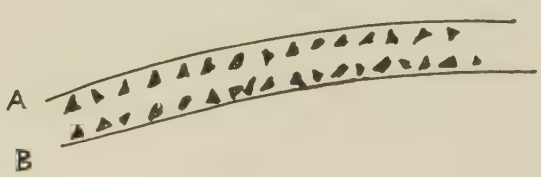

Fig. 1

but only where gradient of I is present. Let us have a similar thing at A, in which the part of I (external stimulus) is now played by $E(A)$. Let for instance (Fig. 1) $E(A)$ be produced in elements which are arranged on a surface A, and let the elements of an adjacent surface B be excited only if there is a gradient of $E(A)$ at the corresponding point of A. Let furthermore the excitation on B be given by

$$e(B) = \text{grad} \log E = \frac{1}{E} \text{grad } E \tag{8}$$

In such a case $E(A)$ and $\lambda E(A)$ will produce an identical excitation pattern on B. Thus we find that on B, which is a part of our system, the same distribution of some physico-chemical quantity is produced, regardless to the *position* and *size* of the external excitation pattern on S, as long as the *geometrical* shape of Γ re-

mains the same. Our system possesses the property of "Gestalt —Transposition."

We may ask ourselves, whether some more complex mathematical condition cannot be imposed on $K(s, s')$ so as already to make $E(A)$ invariant with respect to (5). We may also consider a number of alternate possibilities. All this leads to more technical mathematical problems which shall be dealt with elsewhere.

The excitation pattern at B is a function depending on the shape of the contour Γ'. But this shape can be mathematically characterized in various ways. One characteristic of the geometrical form is the distribution function of all the distances

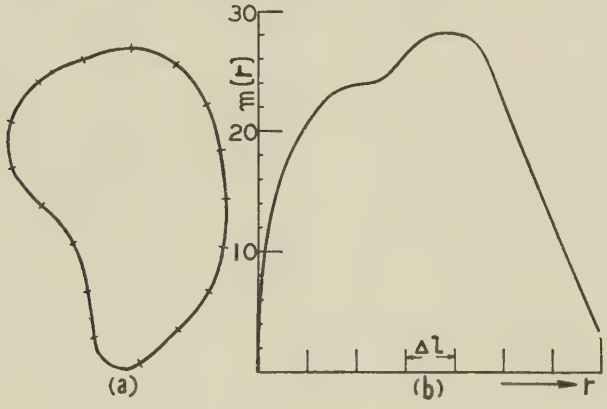

FIG. 2

$r(s, s')$ between any two points of the contour, in other words the *relative frequency* of all the distances $r(s, s')$. Such a distribution curve has been plotted on *Fig. 2b* for the contour *2a*, by taking 17 equidistant points measuring all possible distances between them, and plotting the number of distances, falling in intervals: $l + \Delta l$, against l. Changing the form of *2a* will change the form of *2b*. We may say therefore that the e-pattern at B depends on the shape of the r-function of Γ'. Similar r-functions produce similar patterns.

Let us now consider the two figures of *Fig. 3*. So far as the arrangement of points is concerned, they are very different. And yet their r-functions show a certain similarity. For the *Fig. 4*

the r-function is different. All three r-functions have two groups of maxima, but whereas in those of *Fig. 3* the two groups are approximately of equal size, in *Fig. 4* the second maximum is much more pronounced than the first. Hence there will be a similarity in the $e(B)$ pattern at B for the two cases of *Fig. 3*, but

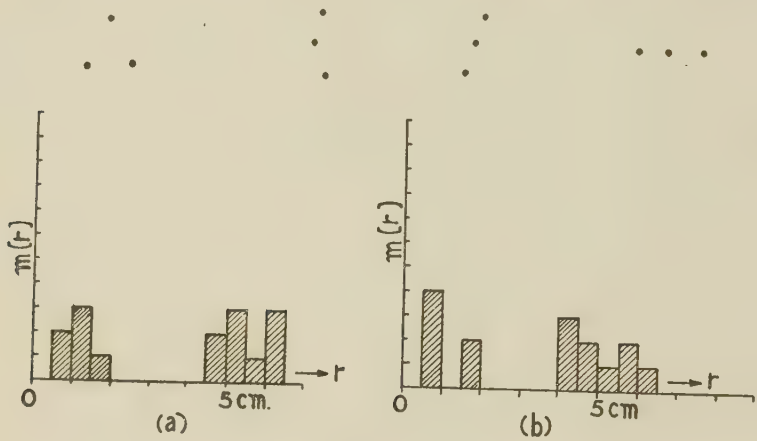

Fig. 3

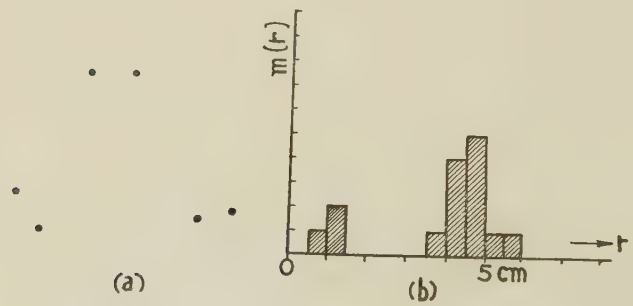

Fig. 4

the pattern produced by *Fig. 4* will be much more different. Subjectively we recognize this similarity by seeing in *Fig. 3 two groups* of *three* points, in *Fig. 4 three groups* of *two* points. The $e(B)$ pattern for the two *Fig. 3* have a great number of elements $\mathfrak{N}^*$ in common and elicit though slightly different, but *similar* responses. We thus find a physical counterpart of "grouping."

Without going here into a detailed investigation of this important problem we venture the suggestion, that the "wholeness" of a gestalt is characterized physically by its r-function. We see "one whole" if the r-function has an absolute maximum near $r = 0$; and the pattern resolves into a greater number of wholes, the more the maximum of the r-function shifts towards larger r's.

We thus see, that such things as the wholeness of a geometrical configuration or the various properties of "grouping" may be quantitatively represented by giving the analytical equation of the corresponding "distribution curve" of the distances. This latter is in its turn closely related to the distribution of $e(B)$ in a definite region of our system. We see thus a possibility of a physical interpretation of wholeness, grouping, etc.

The reader will have noticed, that if we substitute the word "neurone" for "element," the properties 1–7 give a more or less phenomenological description of the central nervous system. We did not use the word "neurone" intentionally, in order to emphasize that we are not proposing here a definite theory of the brain. We are attempting to investigate in abstracto systems which possess the general properties of the brain, just as we have investigated in abstracto systems possessing the general properties of a cell. The problem treated here is a more difficult one. By far not all the possibilities have been outlined and a wide field is opened up for more detailed mathematical investigations, many of them dealing with integral equations. This is an important point. If the characteristic property of the "gestalt" is the functional interrelation of its parts, the proper mathematical tool to deal with it must be the theory of integral equations or more generally functional analysis.[3]

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[3] Paul Levy: Leçons d'analyse fonctionelle. Geuthier-Villars, Paris, 1922.

Logical Analysis of the Psychophysical Problem[1]

A Contribution of the New Positivism

BY

HERBERT FEIGL

THE mind-body problem is—despite appearances—still the inevitable basic issue of unending discussions in recent philosophy. Various types of epistemologies and metaphysics, European and American, have offered their widely divergent "solutions" of the dreaded Cartesian tangle. Is there any hope of reaching a universally acceptable view?

The optimistic approach suggested in the present paper is that of a metaphysically neutral (or, more precisely, strictly ametaphysical) logical analysis of the fundamental concepts. There are, to be sure, acceptable and valuable elements in many of the traditional viewpoints. This, however, becomes clear, not through an attempt at eclectic combination, but through a revision and systematic clarification of the problem itself.

My thesis is simple enough: I claim to show, in this rather condensed outline, that *the strict identity of the "mental" life with certain processes in the "physical" world*, a doctrine accepted by a great number of outstanding scientists and philosophers, past and present, *is not a matter of belief or Weltanschauung* (*dogmatic monism*) *but a truth capable of logical demonstration.* In other words, the Duality of Mind and Matter does not imply two reali-

[1] This paper has been presented, with minor changes, at the meeting of the American Philosophical Association in Chicago, September, 1933.

ties, or two "aspects" of reality, but is merely a duality of languages or conceptual systems. In order to make this thesis and its justification sufficiently comprehensible we shall have to deal first with some very fundamental questions, such as the meanings of "meaning" and the meanings of "reality." Only after a firm position regarding these problems is reached can we proceed to the attack on the Mind-Body-Puzzle.

The analysis to be presented is one of the most important achievements of the new "*Logical Positivism*" (Vienna school) and my report is largely an integration of published and unpublished contributions of *Carnap*, *Schlick*, and *Wittgenstein*.[2] However, the responsibility for this synthesis and its exposition is entirely mine. The purpose of my discussion, besides the clarification of the psychophysical problem, is also to show that logical positivism—still so widely misinterpreted—does not add a new (perhaps "negative") metaphysic to the existing ones but consists merely in the analysis of meanings. Thus it does not solve problems (scientific) but resolves pseudo-problems (metaphysical) by separating the meaningless elements from the legitimate empirical questions, whose positive meaning is thereby clarified.

THE MEANINGS OF "MEANING"

The essential prerequisite and starting point of the following analysis is the definition of "meaning." It is unnecessary for us here to pursue as elaborate a linguistic or psychological analysis as that of Richards and Ogden.[3] For our philosophical purpose the distinction of two fundamentally different types of meaning, the *cognitive* and the non-cognitive or *emotive*, is sufficient. That which signifies or carries the meaning is a symbol or a system of symbols, a language. In the case of emotive meanings we can include—if we wish—a wider range of objects having or conveying this type of "meaning," as in speaking of the meaning of life or the meaning of art. Generally we mean by non-cognitive or emotive meaning the capacity of something to arouse a mental

[2] For a brief account of the main tendencies and results of Logical Positivism see the article by A. E. Blumberg and H. Feigl in Journal of Philosophy, 1931, p. 281.

[3] I. A. Richards and C. K. Ogden, "The Meaning of Meaning," 1923.

picture, an emotion or an action. Accordingly we may distinguish pictorial, emotional and directive or motivational meanings.

Radically different from these are the *cognitive* meanings. It is true, in the actual exchange and communication of meanings that both types are ordinarily present. Yet *logically* it is absolutely indispensable to separate them as clearly as possible. For only the cognitive meanings communicate information, they alone are the vehicles of knowledge. Here a subdivision is essential. We must distinguish formal or structural from factual or empirical meanings. Logical and mathematical knowledge is purely formal, independent in its validity of any concrete applications, whereas all factual knowledge depends upon the test of experience.

It is the question of *factual* meanings which will stand in the foreground of the following discussions. As the criterion of factual meaning we adopt the pragmatic formula of *Charles Sanders Peirce: If and only if assertion and denial of a proposition imply a difference capable of experiential test the proposition has a factual meaning.* (Wittgenstein's characterization of this criterion, although verbally different, is practically identical with Peirce's.) The Logical Positivists have often been misinterpreted on this point. They do not say that a proposition must be *verified* in order to have meaning. Verifi*ability* (or falsifi*ability*) is all that is required. That is to say, they insist that the decision by experiential test must *in principle* be conceivable. Technical or natural difficulties of verification or falsification do not count. Only when by the nature of the proposition a test is logically impossible, we have a factually meaningless sentence.

This definition of meaning is neither artificial nor dogmatic. It is the simple, impartial result of a comprehensive reflection upon how propositions are used in common life and in science. If we are to know what we are talking about we must have an idea under what conditions our statements would be true or false. But if we cannot possibly mention what would have to be the case in order to validate or invalidate our assertions then we cannot distinguish between their truth and their falsity, and that is precisely what we mean by a sentence being "factually meaningless."

Of course, factually meaningless sentences may nevertheless

have important pictorial, emotional or motivational significance. It is regrettable that the term "meaning" is applied to both the cognitive and the non-cognitive functions of language. (If philosophers could make up their minds to some terminological reform it would be very desirable to arrive at a resolution concerning the use of such words as "meaning," "significance," "appeal," "import," "content," "connotation," etc.)

THE MEANING OF "REALITY"

The first step in our analysis of the mind-body problem is the clarification of the concept of *reality*. The application of our definition of meaning leads to the distinction of two meanings of "reality": one cognitive (empirical) the other non-cognitive (emotive). The first is the legitimate concept of existence as used in common life and in the factual sciences. Existential propositions of the empirical type are verifiable in principle. The existence of tables, trees, people, stars, microbes, atoms, cosmic rays, historical events (past or future), mountains on the other side of the moon is capable of test—So also is the existence of perceptions, thoughts, desires and dreams. And although the actual performance of the test is often difficult we all agree that questions concerning the empirical existence of any object of the mentioned types are in principle decidable. We are convinced that after careful observation or experimentation reasonable and sufficiently trained people can arrive at unanimously acceptable results.

The situation is different with the philosophical concept of reality basic in the dispute of the various types of idealism and realism. No agreement has been reached in centuries and although the discussions have grown more subtle and more sophisticated the prospects seem gloomy enough. The whole situation displays a dangerous similarity to the theological one. What is responsible for this condition? Our answer is: Too much concern for argument, too little for meaning. Indeed the concept of reality has never been satisfactorily defined. Many verbalisms have been offered, such as "independent or objective existence," "things existing in their own right," "existence in the sense

in which our own selves exist," etc. All these phrases are ambiguous. They do imply in a way what is contained in the empirical meaning of reality. No doubt, there is a good and verifiable meaning in the idea of independent existence: It is simply not true that a rock springs into being as soon as we look at it and fades out as soon as we turn away. This can be verified by other people, by mirrors, by movie cameras, and so on. But the philosophers are not satisfied with this empirical, scientific meaning. They admit that physicists, chemists, zoologists and psychologists can agree concerning the scientific reality or non-reality of waves, substances, animals, or instincts, but that is—according to idealists as well as realists—not the whole story. There is, they maintain, beyond the questions of empirical and verifiable reality the omninous problem of the "*something more*": the problem whether this scientific reality is "nothing but a logical construction" designed to bring order and predictability into our immediate experience or whether the scientific concepts have "*ontological relevance*," i.e. stand for objects "existing in their own right." This is the famous dilemma of subjective idealism (monadic or solipsistic) and critical realism (monistic or dualistic).

We find an analogous situation in the problem of the "*other mind.*" To ascribe to our fellow men consciousness *in addition* to overt behavior and discoverable physiological processes implies again a transcendence, an introduction of empirically unverifiable elements. The dispute of materialistic behaviorists and animistic introspectionists in regard to this problem is just as hopeless as the struggle of realists and idealists regarding the problem of the external world. We propose that something should be done about this situation in philosophy. The policy of shifting and concealing the issue by all kinds of tricks and stunts will no longer do. Enlightened minds, especially scientists, and scientist-philosophers like Mach, Einstein, Bridgman and others have clearly seen the futility of those problems. Among the philosophers in this country E. A. Singer deserves highest credit for his early critique of the Mind-Behavior problem, which he advanced even before the general battle over behaviorism

started.[4] Regarding the often-referred-to automatic sweetheart, which according to the supposition is physically in every respect indistinguishable from a real sweetheart (with a soul!) Singer is right over against James. He has been completely misunderstood by all his opponents (D. S. Miller, Lovejoy, even M. R. Cohen, et al.) The Peirce-Wittgenstein criterion of meaning acts here like Occam's razor: An unverifiable difference is no difference at all.

THE PSEUDO-PROBLEMS OF REALITY

The unanimous contention of the Viennese positivists concerning these two problems of reality is that they are, in their traditional form, pseudo-problems. They are avoidable if we are in earnest with our analysis of meaning. That peculiar "plus," that "something more" which, it appeared, is needed in order to endow "mere logical constructions" with ontological reach, and "mere bodily behavior" with psychological relevance is factually-meaningless. By the very nature of the situation assertion and denial of the reality of the external world, or of the other mind (if meant in this "additional," transcending sense) are absolutely inconsequential with respect to the verifiable facts.

When we Positivists criticize metaphysics we refer primarily to this type of reasoning which pretends to express assertions concerning the world but nevertheless utters sentences devoid of factual meaning. Our attitude has often been misconceived. Frequently we have been denounced as subjective idealists, or solipsists. This seemed justified on account of our position regarding the external world. But then the critics were shocked by our behaviorism. So they turned round and we were stigmatized as materialists. The truth is that we are neither the one nor the other. (It would be hard anyway to combine these two extremes.) Our point of view is *strictly neutral.* We refrain from ontology. *We believe we have demonstrated that ontology cannot give information concerning the universe. Only science does.*

Thus we *neither assert nor deny* metaphysical reality. However, this is not an agnostic position. Agnosticism is the conviction that we cannot know the answer to a question. But as

[4] E. A. Singer, Mind as Behavior, 1924.

Wittgenstein says: "For an answer which cannot be expressed (i.e. meaningfully) the question too cannot be expressed." Only when we attempt to transgress the limits of meaningful language do we arrive at insoluble problems.

To repeat: Experience which is in every respect "as if" it were dependent upon an apprehension of a transcendent reality is strictly identical with an experience which is "really" so. Similarly an organism behaving in every way "as if it had a mental life" is simply identical with what we can possibly mean by an organism "really" having mental life. In this sense we can accept most of the assertions of the critical realists. They are easily transformed into scientific, i.e. verifiable statements, or into elucidations concerning the logical constitution of knowledge.

ELIMINATION OF FALSE PRETENSE

But still, it will be objected that although we cannot give to those transcendent assertions a verifiable meaning, they must have a meaning nevertheless, and it is this type of transcendent meaning which is basic for metaphysics—which the deplorable Positivists, apparently by dogmatic decree, try to rule out of court. Perhaps we are blind and cannot see those meanings—but I do not think so. What we do see is that those metaphysical meanings are impure mixtures of legitimate cognitive meanings and emotive meanings which are illegitimately presented as cognitive. We do not in the least deny that those emotive meanings are extremely important elements and instruments even in our intellectual and cognitive activities. No doubt, the genuine physicist or astronomer is inspired in his investigations by the idea of discovering the order of an independent nature. No doubt, the searching psychologist often finds and defines his problems on the basis of the inextricable belief in mental states (an "inner private experience") in his subjects. We do not say that these scientists are wrong. There is too much verifiable meaning in these beliefs to forbid even formulations as dangerous as these. But what we can see beyond those verifiable components is nothing but emotive meaning: pictorial, emotional, motivational.

Easily enough the *image* of two spheres or planes is aroused when we speak of reality as juxtaposed to experience. But this image has no cognitive equivalent except in the juxtaposition of organism and environment; but it is precisely this empirical fact which is not meant by the metaphysical relation of experience and reality.

Similarly the phrase which defines the meaning of objective reality in terms of the unique experience of the existence of our own self conveys (at best) a strong emotional appeal. By the very nature of this definition it is incapable of application: If it is only our own existence which is grasped in this immediate manner, then it is never decidable whether or not other objects or subjects share this ineffable quale. The feel of our own existence may be emotionally "projected" "ejected," or "introjected" and in this way accompany assertions of reality, but it is not a logical constituent of such existential propositions. We have here before us the "intuitive" or "mystical" concept of reality—"thereof one must be silent."

The "belief in reality" is, besides these pictorial and emotional meanings, especially *motivational.* Our resolution to interpret experience in terms of spatio-temporal-causal constructions, to proceed according to the principle of induction, in the sense of an operational maxim,[5] i.e. to rely upon the order and regularities discovered thus far,—this resolution is the practical or pragmatic equivalent of the so-called presupposition of reality which was erroneously considered as theoretical. Correspondingly the belief in the mind of the other person has most important emotional and motivational meaning in our social and ethical life. (May I say here once more that we do not deny the mental life in others, that in a certain sense, as everybody does, we assert it, but that all our efforts are devoted only to make clear what we mean by that assertion.)

THE PROBLEM

By now the reader will impatiently expect a more coherent statement of our position. After all, he will say, here are real

[5] See my article, "The Logical Character of the Principle of Induction, this journal, Vol. I, p. 20.

and intricate problems demanding a thorough and consistent clarification. If factual meaning depends on experiential verifiability, pray, *what is experience* and *whose* experience is referred to? Can we meaningfully assert the reality of the psychical (in the sense of immediate experience) and of the physical (in the sense of events in a spatio-temporal order)? Furthermore: Is immediate experience the subject matter of introspective psychology, and if so, how should we interpret its relation to the physical world, particularly to the physiological processes in nervous systems? *What then is the logical relation of psychology and physics?* Indeed, no respectable philosophy can afford to disregard these questions. Our method of approach consists again in an analysis of meaning.

THE MEANINGS OF "EXPERIENCE"

We start with the concept of "*experience.*" The term "experience" is ambiguous. We consider experience on the one hand as the logical basis and raw material of all factual knowledge and on the other hand as a late product of organic evolution. Experience in the first sense is all-inclusive because any conceivable fact must be expressible in terms of possible patterns of data. This is the fictitious "epistemological subject" of the Neo-Kantians. It is a hypertrophied "stream of consciousness" indefinitely extended in its comprehension and capacity. It is a field of possibilities or potentialities. There is no birth nor death to it. There are no blank spots of sleep or unconsciousness in it. It is the chimera of an observer capable of observing everything at any time, at any place, in any direction. We must make sure not to take it for more than a fiction, a convenient metaphorical "façon de parler." We need it in order to pursue the task of epistemology, in order to answer the question: How do we justify our factual assertions? Logical analysis of the building of empirical knowledge leads back to this groundfloor, the possible patterns of the given. It has been shown by Carnap[6] that any concept of empirical knowledge can be constructed by means of

[6] R. Carnap, Der logische Aufbau der Welt, Berlin, 1928.

purely logical operations on the basis of fundamental elements and relations pertaining to immediate experience.

But what precisely is this realm of immediate experience, this chimerical epistemological subject? It is a good principle to insist upon translating metaphorical descriptions into adequate expression. A field of possibilities is clearly not a reality. It is a logical sphere, a universe of discourse. It is the basic frame of empirical knowledge, an open range of variables, a logical type in a sense (at least) related to Russell's atomic level. It will prove most fruitful to formulate the meaning of "experience" in this epistemological sense as a realm of discourse or more precisely as a universal language of a special syntactical type. It is the language of elementary propositions, the *language of data.*

(I have no space here to discuss the problem of the "data" more thoroughly. Carnap has arrived at a tentative solution which is in principle equivalent to the tenets of E. A. Singer, Loewenberg and C. I. Lewis which are better known in this country. The data which figure in the elementary proposition are—to employ Loewenberg's convenient term—certainly *post-analytic.*)

The question "whose experience?" is clearly meaningless in regard to this concept. The epistemological subject, to use the traditional expression once more, is not to be confused with the biological subject. Only the illegitimate identification of both leads to the insoluble troubles of metaphysical solipsism. (This has been made especially clear by R. Reininger—not a Positivist —in his books: Das Psychophysische Problem; Metaphysik der Wirklichkeit, Wien 1930, resp. 1931.)

Nevertheless there is some justification in designating the language of data as solipsistic. But only in the *methodological* sense, which is metaphysically strictly neutral. The question of reality or irreality is, as we have seen, meaningless on this level. Nevertheless, we would miss entirely our epistemological task of reconstructing the building of knowledge if our data were not the possible experiences of an *individual.* It is a characteristic of the structure of experience that after we build up the higher stories on this basis the owner of the site can be identified. The basis as

such is absolutely impersonal but as soon as the spatio-temporal level of construction is reached we can decide whether it was Mr. A's or Mr. B's experience which was selected at the starting point.

THE PHYSICAL WORLD AND THE BASIS OF KNOWLEDGE

I hope that these remarks suffice to convey an idea regarding what I mean by "the language of data." Our next step is the introduction of the "*language of constructs*." According to Carnap's analysis (which represents an important improvement of the well-known theory of Russell), the physical concepts of the spatio-temporal order are logical constructions of a rather high level of complexity. Accepting this level as another universe of discourse we are going to call it a language of constructs or the "physical language." The important point here is that the physical language is *retranslatable* into the language of data. The verifiable content of a physical assertion is expressible in terms of "if-then-propositions" in the language of data. For example the assertion that the specific gravity of iron is 7.79, finds its meaning in the system of all the possibilities of verification; i.e. in such propositions as: if I take a piece of iron and place it on a scale, and measure its volume etc., etc.—then I shall read the pointer thus and so. But the concepts of iron, scale, volume, the operations of measurement etc., are still highly complex and can be further analyzed in their turn until we land at the elementary level of data. (To those who have no sympathy with the technicalities of modern logic or find Carnap's exposition difficult I recommend a study of C. I. Lewis'[7] excellent exposition which leads to the same general result.)

From the *logical point of view*, then, physical reality is a *construction*. But this does not mean that it is a *fiction*. Nor does it mean that it is *only* a logical construction. We are aware of the dangers of the reductive fallacy. Whatever else it may mean it does mean that within the context of logical analysis physical reality is to be characterized as a logical device introduced for the sake of convenience. It is so enormously more expedient to express what we call "law and order of nature" in the language of

[7] C. I. Lewis, Mind and the world-order, 1929, esp. chapters II, III, IV, V and VI.

spatio-temporal constructs than in terms of the terribly cumbersome language of data. The marvelous fact, however, that such a thoroughgoing and beautiful order emerges at all—and already after a relatively small number of logical steps—cannot be viewed as a mere result of our constructive operations. It is a feature inherent but not explicit in the experiential basis. Here the metaphysicians will protest. They will say that what appears according to our theory as a gift from heaven is explicable on the grounds of their theories: The order hidden in immediate experience is derived from its causal relationship to extramental events. Quite so—but that is the *biological* concept of experience and the statement is not metaphysical at all but scientific. Once the scientific level of construction is adopted it is perfectly legitimate to speak of "subject and object"—although perhaps "organism and environment" would be less ambiguous. And the marvel of natural law and order remains at any rate unexplained. Much of what metaphysicians attempt to express in ontology can be transformed into scientific propositions. So when it is said that Reality is not a construction but an object of transcendent inference, again we can give this a good verifiable meaning. Inductive inferences and generalizations are of course employed all the way through in establishing physical laws and concepts but always in the legitimate sense that they are retranslatable into generalizations concerning possible experiences. What in the ontological idiom is called the inference of transcendent objects must be interpreted in this perfectly positivistic way.

THE MAJOR CONFUSION

And here I am coming to a very important point. What we must never forget is to take a definite stand. Either we pursue the task of *philosophy* i.e. *logical analysis*, or we are engaged in *science*. In the first case we are concerned with the reconstruction of empirical knowledge in terms of immediate experience. Ontological transcendence has no meaning here. Everything is immanent, the only sense in which we can speak of transcendence means reference to possible further experience.

In the second case we have accepted the level of spatio-tem-

poral constructs (which is already involved in the common-sense world view) as our universe of discourse; and transcendence means again something quite harmless, namely reference to objects outside of our skin.

Only the confusion of both points of view is fatal. Ontological transcendence is a monster incapable of logical existence. Traditional epistemology has too long suffered from its own hybrid nature. It consisted of *logic* of knowledge compounded with *psychology* and *biology* of knowledge. The distinction and separation advocated and attempted by many schools, as the Neo-Kantians, the Phenomenologists, the Neo-realists (including Moore and Russell), has never been fully achieved. Metaphysics which results from the impure mixture of both viewpoints is not only superfluous, it is strictly nonsensical.

INTROSPECTIVE PSYCHOLOGY AND INDIVIDUAL EXPERIENCE

These insights, correctly understood, contain already the solution of the psychophysical problem. To make this clear I must say a few words about psychology. Descriptive or introspective psychology deals with immediate experience, but not in the way that logical analysis does. Nevertheless introspective psychology employs the language of data. (Again I cannot dwell on the interesting question whether the data here are the elements of the Titchenerian structural psychology or the configurations, patterns and organic wholes of Gestalt psychology. To be sure, neither of them deals with pre-analytic data.) Now as Psychology is not concerned with a logical study of possibilities but with the description of the actualities of experience the question "*whose* experience?" is meaningful and inescapable. Even if psychological laws express regularities of any individual experience, it is the experience of *individuals* which is referred to. Granting this, there is no other possibility of speaking meaningfully of individuals but in the spatio-temporal sense. The individuals are the bodies in their spatial and temporal separations. Philosophy cannot dictate what psychology ought to be, still less to believe what psychologists themselves say about their science, (that is oftentimes very prejudiced)—but has to study and clarify what

they *do* in their actual research work. And here we find that they (I am still referring to the introspectionists) use the language of data, combined with the language of constructs. They speak about experiences but also about the physical situations in which they occur. This heterogeneous type of language is not dangerous if employed with caution. But that seems a difficult task. As we shall see, the outstanding paradoxes and difficulties of the psycho-physical problem result from the *careless* use of this mixed language.

THE TWO LANGUAGES

In order to eliminate radically these logical pitfalls it would be desirable—for philosophical purposes at least—to use only homogeneous language. One possibility we already know: the language of data is capable of expressing everything, if we disregard expedience and simplicity. The physical situations, (environmental stimuli, organic processes, etc.) referred to by the psychologist are retranslatable into the language of experience. Then everything is on a par, the difference between psychology and physics has disappeared; we are in the well-known "impressionistic atmosphere" of empirio-criticism or radical empiricism. Our universe of discourse is the open field of possible data, and on this plane the questions concerning the existence of the mental or the physical world are flatly impossible. (Existential propositions must always have a reference to such a universe of discourse and the only meaningful existential propositions express the presence of a datum characterised by a propositional function.)

Now the advantage of the language of data is its universality: Every verifiable or falsifiable proposition is expressible in it. But this is combined with a grave disadvantage: its subjectivity. It is the language that *I* understand, the language to which *I* have to resort in order to make clear to myself the ultimate meaning of of any factual statements. But without further inquiry I cannot assume that this language is meaningful to anybody else. We are here apparently confined to lifelong soliloquy. For the intersubjective language that we all understand, and which has grown through social interaction and coöperation, is of course the lan-

guage of the spatio-temporal world, the physical language. Regarding the logical problem as to how the transition is made from the subjective to the intersubjective language and vice versa, I know of no better contributions than those of Schlick, Carnap, Wittgenstein, and C. I. Lewis. The essential point in their solution of this problem (seen already by Poincaré, Russell, and more recently nicely expounded by Eddington) is that in communication as already in symbolization only forms, logical structures are expressed, the qualia themselves can be had, lived through, appreciated, but not conveyed. They are ineffable, not because we have no means to describe them, but because *description by its nature is relational.* The content never enters description, for it is unique, and language can at best point to the unique but never express or transmit it. What is accessible is the pattern, the skeleton, the structural system, the form.

AN INESCAPABLE ASPECT OF BEHAVIORISM

This explains why we can understand each other without being each other. But understanding is not enough. Understanding means to grasp a structure. We want more. We want to know whether what the other one says is true. With respect to the common physical world there is no essential difficulty. But in regard to the so-called "experiences of the other one" the situation seems still problematic to the majority of philosophers. But here we must remember the criterion of meaning. What is not verifiable intersubjectively cannot appear meaningfully in the intersubjective language. But fortunately all indications point to the fact that there is nothing that in principle would escape intersubjective verification. If the "other one" tells us that he is hungry, or sees green, thinks of his friend, desires love, is angry or that he dreamt of white elephants we don't have to believe it. We can check it up. (Sometimes we may even know better than he himself does.) It may be objected that this is possible only for the simpler and lower types of experience; that it presupposes a quite utopian state of behavioristic and physiological psychology. I grant the technical difficulties; they are often enormous. It may be practically hopeless to find out with

certainty whether a person reporting his thoughts or dreams is telling the truth. But even in common life we often achieve considerable probabilities. Meteorologists can do no better in regard to barometers and the weather. On the intersubjective level all we can do is to get hold of all available facts of behavior (language-behavior included, of course) and to work in the direction of a causal scheme in terms of which prediction becomes more and more efficient. If this ideal is reached, and to the extent as it is already today, we can say we have knowledge of the "other mind." There is no more to be had so far as cognitive meaning goes. The rest is emotive. In order to do more we would have to *be* the other. Yet, it seems to us there is no reason whatever to deny that physiological research is in principle capable of solving the problem for all kinds of psychological facts.

PHYSICALISM AS A SCIENTIFIC PROGRAM

No philosopher has yet proved that this heuristic program of all reasonable psychologists is limited by insurmountable fundamental difficulties. The dualistic animists and interactionists, just as the vitalists in biology, have failed to prove anything. Their contention, if meaningful at all, very carefully reinterpreted, has perhaps a chance of being legitimately expressible in the language of data. But on the intersubjective level it would boil down to the assertion of a very peculiar, quite miraculous, practically unacceptable type of indeterminism in the behavior of human organisms.

If behaviorism, or better, physicalism, is understood as a methodological program of science, and if this program can be carried out—which, I must really say, seems incredible only to theologians or philosophers who cannot free themselves from suppressed or disguised prejudices, then we are faced with a very salutary situation. Then physical language as employed by all natural sciences is not only intersubjective but also universal, capable of expressing every conceivable fact. Then physical language is not only translatable into the language of data but also vice versa.

This mutual translatability means more than the mere reversibility of the reduction of the physical to the experiential language. It involves a consequence of paramount significance: *The whole of introspective psychology as expressed in the language of data can then be considered as a part of the physical language.* The result of this is not only that a system of hypothetical experiential propositions corresponds to a single physical proposition; but if physicalism is correct, every singular experiential proposition is also translatable into a complex physical proposition. To every proposition describing introspectively what, as we say, is given as a datum of my consciousness, there would be a corresponding proposition in physical language describing, as we say, the condition of my nervous system. From the intersubjective point of view these two types of proposition are only verbally different.

THE DOUBLE-LANGUAGE VIEW

As we have seen above the introspective statement is purely structural. The qualitative content in its lived-through immediacy is inexpressible. What the propositions refer to is said by themselves. We cannot say it for them once more. Mutual translatability means nothing but identity of structure. Logically mutual translatability, isomorphism, means simply *identity* of the two propositions. They seem to have different meanings only because of the pictorial connotations. We are only too apt to imagine on the one hand an entirely private, floating experience and on the other a brain or a system of atoms. It is true, these images have little in common. But images are not concepts; and it is the *concepts* only which are of logical relevance. The concept of a brain-event is a mere structural affair just as well as the concept of an experiential event. An analogy will perhaps help to clarify this point. The qualitative contents of our visual and tactual experiences of an object are certainly incomparably different. But we speak nevertheless of *one* object simply because there is a structural correspondence between the two fields. It is a very promising feature of Gestalt psychology that it attempts the discovery of isomorphous physiological and

introspective patterns. Thus we land essentially at the identity theory proclaimed in its various metaphysical forms from Spinoza and Leibnitz down to Fechner, Clifford, Paulsen, Riehl, many critical realists like Drake, Strong, Sellars, also Eddington, and by such psychologists as Ebbinghaus, M. Prince, Warren, Köhler, Troland, and most recently Boring.[8] However, all these thinkers have had to make use of metaphorical phrases in order to express the nature of that identity: the double aspect theory with its "inside" and "outside" views; the use of such ideas as substance and appearance, mind-stuff and external pattern, qualitative dimension or inner field of physical reality, etc.—but none of them can be taken literally. They are metaphysical because of their pretense of factuality despite their purely pictorial meanings. Nevertheless monistic critical realism already works here with a minimum of metaphysics. But we have to translate it into a language which is not metaphorical, a language whose meaning is perhaps less picturesque but adequate. What we need is an accurate "*language about languages.*" There we can express what the realists mean by the identity of the mental and the physical. There we find the only non-metaphorical, non-metaphysical formulation of the double aspect or double knowledge theory: it is the formulation and detailed analysis of the mutual translatability of two universal languages. This, then, is the new positivistic view of the psycho-physical relation. Carnap—in his recent work on the logical syntax of language[9] has shown in what sense a "language about languages" is possible (this involves a correction of an earlier Wittgensteinian dogma), and has laid the foundation for a most general science of symbolisms. The full recognition of the fact that philosophy is logical analysis in this sense will lead—at last—to a clear distinction between its task and the task of science. In the end we can do without metaphysics. It seems to serve only as an all too human preparation for either science or logical analysis.

[8] E. G. Boring, The physical dimensions of consciousness, 1933.

[9] R. Carnap, Die logische Syntax der Sprache, Vienna, 1934.

PHYSICALISM IS NOT A UTOPIA

The value of our analysis of the psycho-physical relation is, of course, dependent upon the realizability of the physicalistic program. But whatever the fate of physicalism may be, and how little we may ever know about brain-physiology, etc., so much is clear: So far as psychology is to be more than a mere contemplation of ineffable qualia, so far as psychological knowledge can be communicated and verified intersubjectively, it is not only expressible in behavioristic physical language, but its very meaning and method is that of a natural science. The term "behaviorism" unfortunately, has many meanings, but if it means this essentially intersubjective character of scientific psychology it is unassailable. With all this we do not mean, of course, to exclude the use of introspective technique or of introspective language. Introspective terms are often more convenient because they are richer in customary cognitive and emotive connotations and are mostly simpler than the extremely circumstantial and complicated behavioristic descriptions. But if they are used in intersubjective discourse they stand for regularities of behavior or they refer to physiological processes whose detailed nature is still unknown. This is not essentially different from the state of affairs in the older physics, e.g., the laws of magnetism expressed the behavior of certain metals. Thus several ways were known in which metals could be magnetized. The noun "magnetism" stood for that unknown micro-condition of magnetic behavior which only recently has been identified as a peculiar state of the electrons within the atoms. But before this was found, the concept of magnetism was, although less definite and less useful, perfectly legitimate.

Analogously where we use introspective terms in psychology like sensation, memory, feeling, desire, they mean on the intersubjective level those still rather obscure physiological processes and conditions which, if identifiable in the future would make behaviorism, which is so far still a macro-science, causally complete. The micro-laws of psychology will then be simply identical with the physiological laws of complex organic-nervous processes.

THE SOLUTION PROPOSED AND A TYPICAL OBJECTION ANSWERED

And now let me proceed to the conclusion. The solution proposed for the psychophysical problem we have classified as a "double-language view." But this slogan would not mean anything if we ignored the specific logical situation which we outlined before. Psychology as well as physics is expressible in either of the two universal languages. What we must avoid by all means are careless combinations of both. Although intertranslatable the two languages have incompatible syntaxes. It is true, they do have ultimately the same meaning, they speak about the same things; but they organize them in so utterly different ways that they are structurally completely disparate. A material object, considered as a logical construction, translates itself into a complex system of laws concerning manifold connections of data. An immediate experience, considered intersubjectively, translates itself into the description of a complex nervous process within an individual organism.

A typical criticism of this solution maintains that the metaphysical dualism is merely transferred to the admitted dualism of the languages. And why, so the critic asks, would there be these two languages were it not for the fact that they "represent" or "refer to" two separate worlds, or, at least, to two distinct "aspects" of one world? Our answer is: There is an indefinite number of possible levels of conceptual construction, an indefinite number of languages, and a good many of them are actually used in daily life and in the various sciences. The exceptional characteristics of these two languages, however, have a twofold explanation: On the one hand, they are *epistemologically* distinguished, in that the psychological language represents the extreme of immediacy and the physical language marks the first level of an intersubjective order. On the other hand, they stand out *naturally*, because the one language expresses the responses of an individual to his own intra-organismic processes (introspection) and the other the conceptually highly organized account of his responses to his "extra-dermal" environment, including his

fellow human beings (knowledge, on perceptual basis, of the "physical" world).—This shows sufficiently that the divergence of the two languages is not due to a fundamental disparity of subject-matter but lies entirely in the difference of their functions and origins.

The only radical dualism that we acknowledge is the dualism of "form" and "content." But this distinction is the result of a very important but nevertheless artificial, philosophical abstraction. It does not in the least imply a dualism of aspects or of realities. The experienced qualitative contents of my "consciousness" are indeed nothing but the "content" of that "form" which conceptually characterizes my cortical processes.—Any other view of the psycho-physical relation employs either factually meaningless ideas, or else, gratuitous hypotheses.

HOW TO AVOID THE TRADITIONAL PARADOXES

The proper understanding of the suggested *dis-solution* of the problem should make it impossible to relapse into the bafflement over the "insoluble riddle of the universe." If the game of language is played according to the self-established rules of translation nothing of the sort can happen. But if these rules are not sufficiently respected we land in the hopeless paradoxes of the traditional Mind-Matter-problem. Thus if you ask: "Where is the color perceived on the paper before me?" you get into the well-known troubles of percepts or "sensa" as either projected into outer space or as dwelling under your hat. Both alternatives are inacceptable because they are nonsensical with regard to physical space. The projection theory furthermore, is simply wrong from the viewpoint of introspection. The question has meaning only if asked in either one of two ways: First, as a question of descriptive psychology; then the color is precisely where I see it and nowhere else. The correct terminology to express this is the language of data and the space concerned is visual psychological space. Second, as a question regarding the physical processes relevant to the perception of a color. The answer to this question is the old story of matter, light-reflection, nervous excitation and conduction, brain-process, etc. All this is happening in

physical space, which logically analyzed is a construction and correlation of a manifold of data given in the various psychological spaces.

We find the same situation in the other ominous mind-body perplexities: "A motion became a feeling." This again is a horrible confusion of the two languages. No wonder that this formulation appears nonsensical to every careful thinker. The origin of consciousness is a meaningful problem only within the *physical* realm of discourse. There it means the extremely interesting and important scientific problem as to how in the process of physical and biological evolution those highly organized and centralized types of organic behavior (including language and social behavior) could arise, i.e. through what kind of natural laws they might be accounted for. But there is no origin of consciousness in the realm of data. Sleep or prenatal unconsciousness is a behavioristic concept, and so is death. It cannot appear legitimately in a purely introspective psychology.

Also the questions, *monism* or *dualism*, *parallelism* or *interactionism* have to be either discarded altogether, or thoroughly reinterpreted. Our solution is monistic and parallelistic in that it teaches the complete intertranslatability of the two languages. It is dualistic in pointing out the incongruence of their syntaxes. But we have no use for interactionism, except if it is interpreted in its original naïve sense, namely, that our will moves our limbs (but not our brain!), and that a peripheral (but not a central!) nervous process produces a sensation. It is true, these assertions, as they stand, are syntactically heterogeneous and therefore illegitimate. But it is not difficult to transform them into homogeneous language.

The re-interpretation that we would have to introduce in order to isolate the justified elements in other types of metaphysics would do them too much violence. So let us frankly confess that we deem, e.g. Animism, Spiritualism and Panpsychism just as well as Materialism strictly meaningless if they pretend to express factual hypotheses.

SUMMARY

(1) A clear distinction must be made between the metaphysical and the empirical concept of *reality*. The metaphysical concept is unverifiable, on closer analysis: ineffable. The empirical concept stands for the truth of existential *propositions* referring to a definite universe of discourse. There is no significant assertion of reality in science which does not have this form. And many metaphysical assertions (such as that of the independent existence of the object of perception, etc.) can be "empirically turned" and then expressed as verifiable propositions of the scientific existential type.

(2) The "mental" and the "physical" are not two distinct realities but two "logical spheres," "universes of discourse," characterized by their logical syntax. They furnish the formal frame for more specific assertions. Logically they are symbolic systems, languages.

(3) Thus we discern and distinguish (among other possibilities) two important universal languages: the language of immediate experience (data) and the language of constructs (spatio-temporal events), i.e. the conceptual systems of descriptive (introspective) psychology and of natural science. (Psychological and physical language.)

(4) If *psychology* is to be communicable (intersubjectively verifiable), its assertions must be expressible in physical language (i.e. methodological behaviorism and the justification of the unmetaphysical component of materialism).

(5) If *physics* is to be verifiable, its assertions must be expressible in the language of data (i.e. methodological solipsism; the justified element in subjective idealism, empirio-criticism, radical empiricism, etc.).

(6) Every "fact," then, can be represented in either language. There is complete intertranslatability (i.e. the legitimate meaning of psycho-physical parallelism, the "identity" and "double knowledge" theories of critical realism). Apparent difficulties in this solution are met by reflection upon the structural character of *all* knowledge.

(7) Both languages have their definite advantages. The language of data serves as the basis of "meaning" for all empirical knowledge. It also has the advantage of "familiarity." Therefore it is preferred in Psychology and in the cultural sciences. Physical language expresses most expediently the law and order of "Nature."

(8) The traditional difficulties of the psychophysical problem arise out of confusions of both languages. Questions such as those regarding the localization of sensation, the origin of consciousness, the nature of the connection between experience and nervous processes are pseudo-problems. It can be proved that these questions (in their traditional sense) are illegitimate precisely because they violate the logical syntax of our language similar to violations of Russell's theory of types. The proof is not based on arbitrary or artificial presuppositions concerning the nature of "language" but on the use that we all make of our concepts in psychological and physical questions.

(9) The question arises as to whether or not these syntactical traits of language reflect a certain "ontological structure" of "reality." The answer is that what metaphysicians mean by ontological structure can be expressed legitimately only through a logical characterization of the way in which "reality" is symbolized in our language. There is a widespread misconception according to which logical positivism is preoccupied with language and symbols but forgets about the facts symbolized. Yet it should be clear that *being* or *handling* the facts is neither science nor philosophy. It is through symbols that we represent facts, and it is through a study of how symbols "symbolize" that we recognize and can express in the only admissible and fruitful way the so called "metaphysical" truths. The existence of "mind" cannot be asserted in the language of data nor can we express the reality of the material world in physical language. (Only the presence of data or the existence of objects can be formulated.) But what those metaphysical assertions mean appears as a syntactical trait in the two languages. The reality of the physical world means the constructibility of an intersubjective spatio-temporal-causal order. The existence of minds in the world

shows itself in the structural identity (isomorphism) of the basis of introspective language with a certain structure in physical language, called "my nervous processes."

(10) If this analysis leaves a "riddle" unsolved, then it is an insoluble one that cannot be expressed as a significant question in any logically legitimate language. The uniqueness of immediate experience, the uniqueness of the "I," the "now," and the "here," can be contemplated, mystically appreciated, but not conceptualized because these are precisely the things "whereof one cannot speak." Speaking, i.e. conceptualizing, means relating, comparing, classifying, ordering,—and after the unique has been subjected to this treatment it is no longer unique.

If this outline of an analysis proves essentially correct, there is a definite hope of transforming the old puzzle of the psychophysical relation on the one side into fruitful scientific questions and on the other into a field for the most fascinating logical investigations.

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NOTE

Mr. C. I. Lewis' interesting critique of the Positivist definition of "meaning" (in his article "Experience and Meaning" in Philos. Rev., 1934, p. 125 ff.) deserves a more explicit discussion than I could possibly give in this paper. I hope, however, to have answered part of his criticisms implicitly.

As the new positivistic literature is widely scattered I add a few references to relevant articles and books *not mentioned in the text*:

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The Parallel Development of Method in Physics and Psychology

BY

CHARLES HARTSHORNE

THERE is a significant and a too little noted analogy between the histories of the physical and the "mental" sciences—especially psychology—an analogy capable of throwing much light upon the present-day problems of the latter. Some aspects of this analogy have become commonplaces, but others have been almost completely overlooked.

The natural and correct movement of thought is to begin with the most obvious, simple, and seemingly intelligible facts and ideas. Thus physics and metaphysics began together with the consideration of such obviousnesses as fire, earth, water, space, motion, and change. Parmenides introduced the further obvious abstraction of being or existence. Democritus interpreted this concept in terms of "things" in the plain sense of bodies analogous to those which can in the literal sense be handled. For much more than two millennia this remained the dominant concept in physics. The result was the attempt to interpret such phenomena as light, heat, and other forms of radiation in terms of the motion of inert bits of substance. Solid body, the plain obvious variety of thing, was to be the explanation of all things; in the extreme case, even of mind.

In the meantime, psychology—which, like physics, was for a long time not sharply separated from philosophy—adopted a

similar procedure. It, too, even where it eschewed crude materialism, began with the obvious, and attempted to reduce the elusive and the subtle to mere matters of detail or complexity. Now nothing could be more glaringly manifest than the occurrence of red, blue, sweet, bitter, warm, and the like. These sensory contents, as such, could not but be discovered the moment philosophic doubt of the truth of inherited metaphysical beliefs enforced the distinction between appearance and reality. On the other hand, the ideas of value, of beauty, of personality, of emotional tonalities and attitudes, were beset with the most elusive implications, the subtlest shades of meaning, and bristled with obvious difficulties of ambiguity, subjectivity, and complexity of import. Naturally, then, the sensory contents as such, the redness identified merely as "this given red," took precedence in theory over such a matter as the aesthetic interpretation of red, or such a matter as love or hate. Philosophers thus attempted to complete the picture of the world arising from physics by adding to the primary qualities the secondary, the latter considered usually as "subjective" but indispensable signs of the former, and by adding to the secondary qualities the notion of a mind essentially characterized as the mere registration or "awareness" of such qualities.

Values, emotions, enjoyments, sufferings, were secondary, or even "tertiary" phenomena, merely an additional and cognitively less important class of data, or a more special mode of awareness, while the central fact was mind as awareness plus primary-secondary qualities supposed to be stripped of all emotional content. The sensory qualities were the manageable "things," the atoms of psychology, and the mind as the medium of givenness was the space "in" which they had their being. All else, as with the corresponding notions in physics, was a matter of detail or of special complexity. (Often "ideas" were added, but these were more or less atomistically conceived, as in Descartes and Leibniz.) There were, however, the most significant differences between the thinking of the physicists and that of the philosophical psychologists, differences which I venture to express paradoxically but not, I think, perversely by saying that the physi-

cists were, to a certain extent, and not exactly by conscious choice, more philosophical in their approach than the philosophers. The physicists attempted, indeed, to reduce all things to the obvious; but they worked with ideas which *implied the transition* to a subtler or more generalized standpoint more clearly than did the corresponding psychological and philosophical concepts.

The major implication of the kind referred to is found in the physicists' concept of space. For this concept contained in itself that of the *unity* of being, in a mathematically definitive form, while the philosophers' "mind" or "substance" or "being" contained it only in an elusive or ill-defined sense. The physicists possessed, as we shall see, presently, an exact though incompletely generalized logic of likeness and difference, whereas the philosophers oscillated between a crude and false logic of absolute dichotomies and a rather vague, ambiguous, and difficult logic of monism or "organic unity" (expressed, for example, in Hegel). Thus, while the physicists were led centuries ago to formulate the interrelations between particles compresent in space in the triumphantly definite terms of Newton's laws, the vague and far from accurately formulated generalization of the association of ideas remained until some fifty years ago the chief philosophical or psychological law consequent upon the compresence of ideas in consciousness. About 1890, however, Charles Peirce formulated his "synechistic" principle, or "law of continuity," which established for mental elements almost as definitive and universal a system of relationships as was represented for physical elements by spatial geometry.[1] This formulation (the purport of which will be indicated below) was to some degree anticipated by Parmenides (being is a continuum—a sphere without breaks), Heracleitos (the unity of opposites), Plato (the ideas participate in one another), Leibniz (the law of continuity), Kant (the principle of affinity), Hegel (ideas pass into one another, truth is the union of opposites), and doubtless others. Nevertheless, by virtue of its unusually explicit and fully generalized character as

[1] C. S. Peirce, *Collected Papers* (Cambridge, Mass., 1931), 1. 170, and p. 362. Also Vol. 6. And see my article, "Continuity, the Form of Forms, in Charles Peirce," *The Monist*, 1929, pp. 521–534.

presented by Peirce, it was one of the signal achievements of the human mind. It represented the fulfillment of the ancient Platonic dream of a genuine welding of mathematics and philosophy. Owing to the non-publication of most of the writings in question, and to other circumstances, philosophers have paid scant attention to this proposal of Peirce; but the general movement of thought appears to be somewhat in its favor. A minor example of this is the independent appearance of substantially the same conception in the minds of several thinkers of the present day.[2]

The Law of Continuity proclaims that geometry is the key not only to relations in space and time but *to all relations of likeness and difference.* This is really implicit in the physical conception of space-time as a unity embracing in itself all the diversity of existence. The progress of physics has consisted in no small measure in rendering this implication explicit and specific. Some of the stages of this reduction of differences to the form required by the law of continuity may be sketched as follows. In the nineteenth century the difference between material elements was traced to different arrangements of parts, and in the twentieth century these parts have been shown to be of a very few kinds, distinguished from each other only by quantitative properties such as mass, or by the dimensional polarity of positive or negative charge. Heat had long ago been interpreted as merely a mode of motion. Finally the diversity of light from electric phenomena, and of both from solid matter, were similarly relativized. On the one hand, the attempt to explain light by the obvious concept of movable bodies was shown at last to be impossible; while on the other hand the work of Faraday, Maxwell, Kelvin, the Curies, Planck, etc. showed that the concept of wavilinear radiation could be used to describe not only light and electricity but also even solid matter, and indeed that such description of the latter was unavoidable. All differences, except the polar one of positive-negative, become thus radically quantized as the principle of continuity demands. Also the plain straightforward concept of movable objects gives way to the less "tangible" conception of four-way processes or "events;" that is

[2] The late Ralph Eaton was one of these, according to a conversation I had with him.

to say, entities the spatial character of which cannot be abstracted from the temporal. But if time—and hence change—are thus essential, and are not a mere series of static qualities, then the old question *what* is it that changes, becomes only the more insistent. Physicists now declare frankly and explicitly that at bottom this question is unanswerable from the physical standpoint. Physics ascribes spatio-temporal equations, i.e., systems of relationship, to the qualitatively perhaps unknowable matter or "electricity" or "ether" or "wave-particles" on the one side, or to the known and predictable qualities of human perception on the other. Thus the obvious has yielded to the elusive, the solid and objective to the seemingly intangible or the subjective. What remains? We may answer: the principle of universal relativity or continuity. Conceived as radiant energy, all matter is one and in a sense continuous throughout space, and an individual thing is merely a knot of intersecting spatio-temporal world lines, merely the one world-process drawn to a particular focus. The discontinuity of quanta is not inconsistent with this assertion. The only discontinuities which the Law of Continuity denies are: (a) gaps in a series of possibilities as such; (b) *infinite* or *absolute* gaps between actual things. Now the quantum is finite, it has a measure, it might have been greater or less, it is not a violation of the Law as (b); and since it does not deal with possible worlds it cannot violate the Law as (a). As for the mode of analysis employed in working out this view, it is no longer the conception of taking to pieces, of isolating absolute parts or elements, but of establishing functional relationships of things which are themselves functions. Nothing in physics is anything but a function save, on the one hand, the data of perception as such, which are, so far as science posits them, functions of human experience taken as a whole, and on the other hand, the physical world as a whole—a problematical concept!

To this development, also, there is a parallel in psychology. For the physical thing, the obvious starting-point of physical inquiry, substitute the sense datum in the interpretation which is criticized in this article. For radiation, energy, process, and functional or organic unity, substitute affective tone, striving, en-

joyment, and finally beauty, which is the felt interdependence of part and whole. These were the elusive phenomena, regarded for two millenia as subsidiary. These were also the aspects of subtle *continuity*, living *process*, organic *interdependence*, in the realm of immediate data. We have seen how matter dissolved into electricity, pervading all space; will the apparently quite "ultimate" sensory qualities turn out, in similar fashion, when subjected to closer inspection, to represent but unusually stable condensations or phases of that affective (action-inciting) tonality which is from the activistic-biological, or legitimately behavioristic standpoint, the essence of mind?

That the change now in process in psychological concepts is genuinely parallel to the development in physics can be seen in the present status of the old doctrine of association. This was a quasi-mechanical affair. Ideas cohered to one another like Newtonian bodies in space, and this relationship was the most ultimate one which occurred. The mere "unity" of mind was just an added fact which, as was true formerly of the unity of space, played no active or intelligible part in the process. According to a recent writer, the supporters of this doctrine represent but a small minority of contemporary psychologists.[3] The point of view which is taking its place holds with Bergson that the interpenetration of contents is the basis of all their relationships. Müller-Freienfels, for example, declares that "the sole phenomenon to which all laws of association are to be referred is that of 'spreading' " (*Ausbreitung*, the very word employed by Peirce in his article, *The Law of Mind*). Ideas, he explains, are not sharply separated from one another, and the degree of their relative distinctness is unstable. Furthermore, "The ground of this phenomenon of spreading is found not in the representations but in the feelings. Wherewith we at once win the advantage that consciousness appears not as an objective mechanism functioning upon the surface of the self, but as a process which the self (that is, *the sum of the affective dispositions*) itself determines."[4] [Italics

[3] Georges Dwelshauvers, *Traité de Psychologie*, p. 389.

[4] Richard Müller-Freienfels, *Das Denken u. d. Phantasie* (Leipzig, 1916), S. 241. Cf. C. S. Peirce, *Collected Works*, Vol. vi, Bk. 1, Ch. 5, or *Chance, Love and Logic*, pp. 203 f., 222 ff.

mine.] Dwelshauvers quotes practically the same thought from Krüger, and develops his own position along similar lines.[5] The subtle but all important and all pervasive factor, analogous to radiation in physics, is feeling.

In drawing this analogy between physics and psychology we must not, however, overlook an important difference. In physics the fundamental entities are inferred rather than immediately discerned. There is, therefore, a semblance of justice in the notion that the qualitative nature of these entities is unknowable. But in psychology, so far as this is construed as the study of the directly given as such, no such agnosticism is plausible. If there is a common factor, analogous to the ether or to space-time, involved in immediacy as such, and capable of constituting the essence, now of sensory, now of emotional, now of intellectual phenomena, this common protean principle must be observable, since it is the essence of all observable entities as such. The final clue to the nature of that continuity which is the rationality of existence must therefore be sought in the psychology or philosophy of immediacy.

On the other hand, the subtler or organic doctrine of mental factors is likely to encounter in philosophy and psychology even stronger opposition than has the corresponding view in physics. For since it is immediate experience which is to decide, it is open to anyone to offer, as a final objection to the hypothesis of affective continuity, the assertion that the *non*-affective character of the sensuously given is itself a datum in his experience. This objection cannot be adequately considered here; but I will remark that such a simple appeal to self-evidence is an example of the general fallacy of observation unguided by initial well-defined hypotheses.

If, for instance, the reader means that certain examples of affective experience, held by him to be typical, differ *widely* from sensuous contents, I grant the truth of this. But if he means that they differ *absolutely*, I deny that the idea of "absolute difference" can be defended against logical criticism. Indeed I hold it to be formally self-contradictory as well as empirically unveri-

[5] *Op. cit.*, p. 408.

fiable. If, then, the reader merely means that what would ordinarily be classed as affective experience, and what would ordinarily be classed as sensuous experience, are *very, extremely* different, I reply that in science all depends upon the how much, and that the mere assertion of an indefinitely wide qualitative gulf between things is a starting point, not a terminus, of thought.

Ptolemaic astronomy asserted such a gulf between terrestrial and celestial bodies, a gulf concerning both their inner nature and the laws of their motion. The next important advance in astronomy consisted, in no inconsiderable degree, in the obliterating of this alleged gulf. Likewise pre-evolutionary biology gloried in the distinctness of classes or species. Lamarck, Wallace, and Darwin were led by the inexorable implications of observed facts to relativize the whole logic of species and genus, to *substitute for the logic of classes and dichotomies the logic of an ideal continuity of forms*, to which within certain quantum limits an actual (genetic) continuity could hypothetically be made to correspond. Just this I would propose for the elementary "classes" of mental contents or functions—and this on the basis of observed facts whose implications are, I believe, likewise inexorable.[6]

The physicist Clerk Maxwell pointed out that the doctrine of determinism arose because, in the more obvious laws with the discovery of which physics began, the statistical character of these and all laws (so far as actually verified) was readily overlooked, whereas the subtler and later discovered laws of molecular and sub-molecular activity brought into unmistakable prominence this very character, and eventually, as Maxwell dimly foresaw, led to the recognition that the older laws owed their deterministic appearance merely to the fact that they dealt with processes in which the "quantum numbers" are very high, in short with in-

[6] I have dealt with this subject in *The Philosophy and Psychology of Sensation* (The University of Chicago Press, 1934). See my paper on "Sense Quality and Feeling Tone," *Proc. of the Oxford (7th) International Congress of Philosophy*, pp. 168–172. A recent statement of the experimental evidence confirming the position is given by Heinz Werner in his article, "l'Unité des sens," *Journal de Psychologie*, XXXI, pp. 190–205. See also the series "Untersuchungen über Empfindung und Empfinden" by Werner, Zietz, and others in *Zeitschrift für Psychologie*, Vol. 114, pp. 152 ff.; Vol. 117, pp. 230 ff.; Vol. 121, pp. 257 ff.; Vol. 125, pp. 249–288; Vol. 127, pp. 265–289.

stances in which irregularities are statistically negligible, or in which probabilities become so great as to simulate certainties. Consideration of the status of the affective factor in present day psychology, in the light of the history of the subject, suggests a similar observation. The most obvious instances of mental contents, such as the physical sensations, are those in which the affective character is most easily overlooked (for reasons which cannot here be discussed). The increasing subtlety of methods, however, brings us nearer and nearer to the point where the general truth can no longer be escaped that, as Whitehead and many other philosophers have suggested, the subtle elusive feeling-tones of experience are the bridge to the understanding of *all* its qualities. Paraphrasing Maxwell, we may say that if what is now known concerning the rôle of emotion, and—we may add—of aesthetic intuition, in the life of the mind, had been discovered prior to the development of the current doctrine of sensation, it is doubtful if that doctrine would ever have appeared plausible.

Physical materialism is on its death bed; psychological materialism is moribund. I suggest that the notion of extra-emotional elements of an at least predominantly emotional experience is quite as baseless and logically objectionable as the nineteenth century concept of solid material particles embedded in, and yet not in their intrinsic nature a manifestation of, the dynamical process of nature as a whole. The life of the social emotions, which is the essential life of man, does not flow merely around sensations, but through them, it is their whole being and content. They are relatively stable and objectified (socially externalized) condensations of the swirl of feeling, somewhat as electrons are described as relatively stable and sharply-localized phases of the activity of space-time.

The crucial problem, however, is methodological. The secret of existence, its spirituality, may be an open one; and indeed thinkers have been aware of it since Hegel, nay, since Plato, but they have lacked an adequate logic for rendering it fully articulate, and a sufficient encouragement from the example of science to gain a hearing, or even to understand their own convictions, in the face of the enormous and apparently hostile influence of

the natural sciences. With this technical lack, and this counter-influence, is it any wonder that agreement has not been reached? But now that science offers to philosophy both a logic and an example favorable to the rational application of the ancient program, the situation is so far altered that one may reasonably say that all pessimistic inferences from the past failures of philosophy to an unpromising future for that study are fallacious, and that the prospects before us are, in sober truth, magnificent almost beyond the dreams of even such professional dreamers as the metaphysicians.

The world-view to which we are likely thus to be led is not the idealism of Berkeley or Hegel, which reduced matter to a set of ideas in the mind of the absolute, but a revised version of the panpsychism of Leibniz and so many other scientifically trained thinkers (e.g., W. K. Clifford), according to which the entities studied by physics have the same nature, though to a vastly inferior degree, as the individuals studied by psychology. The continuity between physics, biology, and psychology—which behaviorists rightly insist upon—must accordingly be read both up and down. An animal is an elaborate "machine" only if a molecule is a very simple "animate" individual. The law of continuity thus opens up a prospect of a physics which will eventually embrace all behavior,[7] and of a psychology which will eventually embrace all individuals in their inner nature, or as they are for themselves.[8] Thus there need be no vitalistic break between matter and life.[9]

[7] For an eminent physiologist's conception of a similar program see Ralph S. Lillie, "The Living and the Non-living," *The American Naturalist*, Vol. 68, pp. 304 ff., and his article in this journal, vol. 1, p. 296.

[8] See *The Philosophy and Psychology of Sensation*, p. 28 and secs. 28, 37–40, for some suggestions as to how the psychic can be translated into behavioristic and therefore publically verifiable and potentially scientific terms. Since this book has been interpreted (see the review in vol. 1, p. 361, of this journal) as an attempt to substitute qualitative for quantitative analysis, it may be permissible to point out that my intention at least was rather to substitute a geometry of qualities for their mere classification, and that since geometry is the foundation of measurement, such a shift could hardly interfere with and ought immeasurably to facilitate the application of quantitative ideas. I should like also to express my sense of the unusual merits and general fairness of Mr. Ruddick's review.

[9] Cf. J. B. S. Haldane's article in the first number of this journal.

The outstanding intellectual fact of our age is the abandonment by those sciences supposed especially to employ it of that "abstract understanding" of absolute dichotomies—such as that between matter and mind—condemned by the great speculative idealists, and the substitution of a mode of relativistic analysis in many of its features akin to the Hegelian dialectic, yet in controllability, in impersonality, immeasurably its superior. The great fact is that we now know that there is a third possibility over and above the dichotomy of dichotomizing analysis versus dialectical synthesis; namely the method of functional (mathematical) empiricism. In other words, we now see that the method Hegel criticized in philosophy is illegitimate even in natural science, although the method he recommended as a substitute is in both science and philosophy no less objectionable and superfluous. In the hands of this third method, which is organic but not dialectical, universal or philosophical in scope, but yet empirical, doing justice to the spiritual unity of experience (its emotional and social character) and yet mathematical and analytic—in the hands of this method lies, I venture to think, the future of philosophy, and of the sciences so far as they are dependent upon philosophy.

It may be well to give an example of the method thus abstractly outlined. Long ago a Greek thinker announced his conviction that qualitative opposites—hot and cold, living and dead, youth and age—were "one and the same." This declaration had the appearance of a mystical insight, or a mystical absurdity, until Hegel endeavored to incorporate it into a logic in which not merely contraries but contradictories—e.g., being and not-being—were one and the same. But this, too, since it conflicted with the working logic of all science, could only be regarded as either a mystical insight or a logical perversity—most probably the latter. But the moment one takes seriously a concept whose significance was indicated by a thinker who likewise lived nearly twenty-five centuries ago (Parmenides), namely the concept of being as without gaps, or as continuous, one sees that so far as this concept applies the identity of opposites is neither a super-logical insight, nor an anti-logical dogma, but simply a mathematical

truism, none the less important for all that. Red and green, blue and orange, black and white—these opposites upon the color solid *must*, by geometrical necessity, have qualitative one-and-the-same-ness. For in any continuous manifold of more than one dimension this identity of opposites must occur. No matter how far apart two points in such a continuum may be, they can be connected by a line, and this line being viewed as a dimension appears as a point upon a dimension at right angles to it, so that with respect to this latter dimension the two points are identical values of the same variable.

Now there is an important application of this principle to the sense of harmony in color combinations, which, since it belongs to aesthetics, might seem quite beyond the reach of exact scientific methods. On the contrary, a flood of light is thrown by the above mathematics upon the preference for approximate complementaries over semi-complementaries. For it is an aesthetic axiom that the more variety the better, so long as unity is not lost, and the more unity the better, so long as variety is maintained. Now although yellow is both different from and similar to green, it is neither *widely* different nor *closely* similar, *both* difference and similarity being therefore merely median. But red and yellowish green are at once strongly contrasted and almost exactly similar, so that both dimensions of value (unity and variety) are close to a maximum at one and the same time. On the other hand, it is best that the colors should not be exactly complementary. For the most subtle results are obtained by varying *all* the factors in an aesthetic situation, by avoiding *exact* identity of any kind (except in the temporal arts where a time lapse softens the monotony, and except where the pattern otherwise suffers from too great complexity); and moreover the maximum possible diversity in any respect means a very violent transition, in so far a "discord," which in a very simple pattern such as a mere color pair can hardly be adequately absorbed by the harmony attained in respect to the other variable. Thus the relationship of precise complementaries is at once too "obvious" (as akin) and too violent or "crude" (as opposite). The best pairs are hence either "analogous" colors (two hues of red)

or else two colors a little over two-thirds of a complete complementary distance apart and hence, by the foregoing geometry, analogous (but not identical or precisely similar) in terms of one variable or dimension, and widely enough different in the other dimension to avoid both the Scylla of an unsuccessful or too remote suggestion of analogy and the Charybdis of too violent dissimilarity, in that dimension. Q.E.D. We have just completed the demonstration of a theorem in geometrical aesthetics. Careful experimental observation and aesthetic wisdom alike verify the conclusion that the most pleasing color pairs are either shades or hues of the same colors or definitely contrasting but less than complementary colors.

Triads or quartets of color present different problems, but the same general assumptions apply. These assumptions of course are not purely geometrical, but neither are those of physics. The foregoing reasoning is a genuine even though simple case of *applied* mathematics. Of course the value of works of art can never be determined by such mathematical principles, since the latter serve at most to distinguish chords from discords, and since significant art works will contain both, as well as numerous other factors, in indescribably complex mixture and integration. It is thus really the complicated and not the elementary or simple that is unanalyzable and inexplicable. Let us not forget that the three-body problem in physics has never been solved. A picture is the simultaneous attraction and repulsion of many thousands of distinguishable specks of color together with manifold representational meanings. Here indeed is mystery, though even here an Albert Barnes can do something to shed light into the darkness, or rather to impart clearness to this blinding glare of light.

But if—as by geometrical necessity must be the case—complementary colors have an aspect of similarity, in what, concretely, does this similarity consist? I have suggested elsewhere that it can be specified in empirically verifiable terms only by reference to those polar contrasts of joy and sorrow, action and passion, intensity and faintness, which apply equally well to experiences ordinarily classed as affective or emotional.[10]

[10] See Hartshorne, *op. cit.*, Secs. 22(3), 28, 32.

Those who oppose such an emotional or aesthetic description of immediacy—which in principle is advocated by Whitehead, Heidegger, Rickert, and many other recent philosophers—are trying to bring to a halt in philosophy a movement which has proved more and more irresistible and fruitful in physics. And what a barren topic the old "epistemology," divorced from aesthetic intuitions and mathematical analysis, has proved to be!

I add that the above indicated explanation of color harmony (which has also application, though in less obvious fashion, to harmony of sounds[11]) has not been set forth in the literature of aesthetics (except in some remarks of Goethe whose significance has not been appreciated). The reason is plain—the neglect of geometrical order, the gateway to all knowledge.

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[11] *Ibid.*, Sec. 33.

A Preamble to an Organismic Theory of Knowledge[1]

BY

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ONE of the interesting and encouraging signs of our times is the fact that the several sciences are learning to develop and carry their own philosophy. Here psychology seems to me to be a bit backward. In this paper I am presuming to suggest that the time has come when the psychologist should take over the subject of theory of knowledge, emancipate it from the leading-strings of Mother Philosophy, and make it a phase of his own science and of its organizing and interpretative philosophy. In fact, I am suggesting that thinking and knowing of whatever sort, even psychologizing and philosophizing, being matters of "mental" activity, are properly object matters for the psychologist, when he really knows his stuff; and I am presuming to say that he is now well enough advanced towards this goal so that he should accept this implied challenge and proceed to take possession of this most significant portion of his proper field. This should greatly enlarge the field of psychological experimentation; it should open up new areas, that are even more significant for our science than are animal psychology and the psychology of learning, as we have hitherto

[1] A paper presented before the Section of Psychology, of the Michigan Academy of Science, held at Ann Arbor, March 16, 1934. For some of the ideas and phraseology used in the paper, the author is indebted to Korzybski's work, *Science and Sanity*, Science Press Printing Company, Lancaster, Pa., 1933.

envisioned these; and it should result in a development of principles for the better organization of our findings in these fields.

In order to get the subject definitely before you, I shall briefly present a more or less pictorial and folk-lore description of knowledge, as I conceive of it; and here I am thinking of knowledge at its best—scientific knowledge, such as is developed by appositely reflective thinking upon some concrete object matter. Our expressions "reflection" and "reflective thinking" are highly descriptive and significant of the processes of knowledge-getting and consequent knowing, or knowledge. In originally empirical and experimental thinking, or trying to think, reflectively, we submit or adjust ourselves to our object matter, in order to get its specifically patterned nature reflected, mirrored, or appositely mapped, in the medium of our experience. Science may thus be described as intellectual map-making. A scientific knowledge-map, for purposes of social communication, must be further re-mapped, in terms of language. Such a knowledge map, made in the language of one linguistic culture, may be still further re-mapped, or translated, in one and another language; but the translator, in the interest of relevant accuracy, must himself in each case possess a foundational knowledge map relatively homologous with that of the original, pre-linguistic, sub-language, or foundational map. And this original may be further abstracted and universalized, in terms of a mathematical language.

The language map, be it noted, is not itself the real knowledge map; nor is this latter the object matter known about. One might commit to memory a verbal knowledge map, and many accurate verbal translations of it, without possessing the actual knowledge map, mirrored or reproduced in terms of these several language maps, and therefore without knowing the objective meanings of his knowledge map. On the other hand, if he has had sufficient observant and reactive experience with the sort of object matter that is thus re-mapped in language terms, he will be able, with the aid of this language map, to organize within himself a comparable knowledge map of the original object matter. It is only in this way that a "philosopher" may reflect the pattern and order of nature, in his work.

Obviously, then, scientific descriptions and organizations of formulae, whether vocal or written, are not themselves the knowledge thus represented. Neither are the sensory patterns effected by contact with the object matter that has been under observation and manipulation. Thus the content of knowledge is not to be found in either the sensory processes of our contacts with our object matters nor in the language reactions that have been devised for denoting and recording the knowledge effects of such contacts. Knowledge is somehow intermediate between sense experience and our speech reactions to it. This will become clearer of understanding when we finally learn that sensations are properly to be located as stimulus-receptor processes, and not as cerebral or synaptic processes, nor "psychical" concomitants of these.[2] The verbally partial or abstract level of knowledge constitutes only the "that" of knowledge—differential linguistic denotations of complex wholes, orders, classes, relations, and individuated items known about. The knowledge "what" or content as such is far deeper and more pervasive of the organism than is either sensation or speech reaction. Just what this content appears to be, we shall consider later. I now proceed to state more in detail my assumptions regarding the knowledge enterprise, its functions, the organismic conditions of its carrying on, the nature of knowledge, and the criteria of its "truth."

But, first, it will be well to indicate the sort of frame of reference within which my own thought moves, in this discussion. For any kind of intelligently rational thought and conduct, there

[2] When someone is actually able to distinguish in immediate experience, temporally or otherwise, between the fact of being stimulated and a fact of sensation correlated with the stimulation, then it will be time to think of sensations, or "sensa," as occurring within the brain. "Sensation" and "stimulation" are two ways of denoting precisely the same fact, where the stimulation is "adequate." Effective stimulation of the foot of a "spinal" man would just as truly be a case of sensation as if the man were normal and could verbally report the fact of stimulation in addition to twitching his foot. As a matter of fact, the word "sensation," as being a traditional mythological holdover from pre-scientific psychology, and thus an inapposite denotation of its designate, should be entirely eliminated from our psychological vocabulary. Similarly with the expressions "primary," "secondary," and "tertiary qualities." These all imply a notion that our receptor cues to action somehow present us with a direct apprehension of some immanent constituent of our stimulus "events." Of such inapposite notions and vocabulary is metaphysics constituted—which scientists tend to mistake for philosophy.

is required a space-time frame of reference within which they are to be oriented and exhibited. In the days of Aristotelian-Euclidean-Newtonian physics, men tried to carry on in a merely three-dimensional, Euclidean or common-sense frame of reference, as the great majority do today—and worst of all, our social scientists, politicians, and statesmen. Yet already, through our developing evolutionary tradition, we have been developing a backward, historical time-perspective, which should be suggestive towards a development of a similarly long future time-perspective, even for human living. This of itself should serve to give individual, institutional, national, and the species life an orientation for thought and conduct that would prompt to far more intelligent and rational action than has ever before been possible. Yet we should be careful not to identify our space-time geometry or frame of reference with the content itself, upon which we use this geometry. We should not even identify it as an invariant space-time that is already given, as being "there." Our astronomers have been helping us to develop a space-time frame of reference with indefinitesimally maximal dimensions. Within this, we develop a smaller space-time frame of reference for considering the environment and nature of the human species —say, our solar system; and within this, a still smaller one, involving our species in its terrestrial habitat. No nation, institution, or even individual can live a fully rational and intelligent life without conceiving of itself within this spatially earth-wide frame of reference, with a temporal axis or co-ordinate of indeterminate and incomprehensible length. Yet consideration of the length of this temporal co-ordinate makes it inconceivably more significant for promoting and giving direction to our life activities than does consideration of any of the spatial co-ordinates and dimensions of this purely human frame of reference. Even in the case of our individually significant acts—our acts leading to personal consummations, the temporal co-ordinate and the temporal segments of our lives are the most significant. This is a fact that the Watsonian behaviorism, with its merely stimulus-response technology, failed to note, but is one which Kantor, with his concept of "behavior segment," does take into account,

in a really significant way.[3] Even Dewey's "complete act of thought" stops short with the factual validation or substantiation of an hypothesis; it stops at just the moment when the organism is properly prepared to go forward observantly or thoughtfully with the practice or technology that would result in a consummation.

It is within such a Chinese-box set of frames of reference that the thought of this paper is carried on.[4] Yet the description of thought, here presented, moves within a frame of reference applied to a processal organism-environment content, and to a segment of such content having the initial and consummatory ends of thought as its temporal boundaries. A scientist cannot even devise and set up an experiment without already "having in mind," in anticipatory, substitutional form, a concept of its conclusion. Our so-called social and political experiments are quite lacking in both a specifiable frame of reference and a concept of their outcomes. Hence it is that such "experiments" are so opportunistic in character and that we have to study autonomous "trends" for purposes of getting our bearings, instead of rationally determining our trends in advance.

Science may be described as a pioneering enterprise of exploring and intellectually mapping the nature of man and of his terrestrial

[3] J. R. Kantor, *A Survey of the Science of Psychology*, The Principia Press, Bloomington, Ind., 1933, pp. 21–27.

[4] Recognition of the importance of thus specifying the frame of reference within which thought and rational action move, I owe to suggestions of my friend, Arthur F. Bentley, of Paoli, Indiana.

A simpler and perhaps more effective way of describing our frame of reference would be to say that it consists of four axes or co-ordinates intersecting in a common point within ourselves. The three spatial co-ordinates thus intersect at right angles with each other. With reference to these we spatially locate objects and events as being before or behind us, to the right or the left of us, and up or down from us—or north or south, east or west, above or below our own spatial location. With reference to these, also, we reckon spatial distances from us. Upon our time co-ordinate we temporally locate events as being before or after our temporal present, and reckon the temporal intervals by which they are thus removed. It is hardly correct to speak of a temporal dimension of anything other than our time co-ordinate, or processal segments of reality as demarcated by their termini with reference to this co-ordinate. If we look at the matter in this way, then we may say that one space-time frame of reference—or set of space-time co-ordinates—will serve us for all purposes: for thinking about things cosmic and terrestrial, and human and personal, and even such limiting cases of behavior segments as reflex acts.

and cosmic habitat, to the end that man as a natural species may make himself intelligently and rationally at home in the natural world which constitutes his environment. Because the territory thus to be explored and mapped is so vast and so complex, it has been progressively parceled out into more or less specific regions or areas, for greater appositeness in the mapping.

Philosophy may be described as the critical and constructive enterprise of general logical supervision over the work of science, to the end that the knowledge map of each field shall be self-consistent and that those of adjacent or closely related fields shall be consistent with each other. This statement of the case implies the assumption that our world is some sort of unity, a real cosmos. As scientists themselves should know their own fields and the history of effort in their respective fields better than any one else, it would seem evident that the several sciences should develop and carry their own respective philosophies and that, by attending to the philosophies of fields closely related to their own, they should also carry on the work of co-operatively integrating the knowledge of their several fields. In other words, science should develop and carry its own philosophy, and there should be no such thing as a strictly or abstractly philosophical enterprise.

All our extra-human sciences, physical and biological, tend to converge upon the conclusion that our world is describable as a natural process, a spatio-temporal world, a world of behavior, tending ever and ever to achieve a further continuance of its own existence, with evolution taking place incidentally and instrumentally to this continuance of existence. And time, in its fundamental reality, is the rate at which this processal existence goes on.

By describing our external world as a spatio-temporal affair, I do not mean that space and time are somehow constituents of its nature. I mean that it is such a world that we have to employ a spatio-temporal frame of reference for purposes of describing and thinking about it. Space and time are matters of our own thinking—are purely imaginary. The act of imagining them is just as truly an organismic act as is any other. "Space" and

"time" are names for our *experiences* of intervals between points, surfaces, events, etc., and thence are semantic substitutes for such experiences. We may think of our temporal frame of reference as a "world tube," within which is our three-dimensional world. Thus we should not say that our external world is four-dimensional, but that our frame of reference is—a very different matter.

The human species, like all the other organic species, may be described as a naturally evolved processal strain within the natural world; and along with these others, it inherits the tendency to continuance of its own existence. Knowing this natural tendency of our species, and, by implication, the function of each successive generation, it becomes our rational business to translate this natural function into purposive living. And thence it becomes our proper business continuously to make over our environment relative to support and continuance of the species life, and to organize and co-operatively exploit ourselves on an earth-wide scale, in the continuous performance of this supreme human function. Thus, incidentally, will our own evolution, individual and social, take place as needed.

Psychology, then, constitutes that phase of the scientific enterprise by which we explore and intellectually map the nature of humanity, socially and individually, as a motilely adaptive strain of the natural reality, relative to its function of temporally continuing its existence. Thus psychology must learn to become a non-Euclidean science—even a science more concerned with the temporal than with the spatial dimensions of its object matters. In fact, only as it discerns its work, like that of medical science, as chiefly a specifying of the conditions and instrumentalities of healthful living—a preparing of the intellectual ground for more intelligent practices and technologies of living, will it be able even to raise and define the really significant problems of its field. It cannot justify its claim to be a biological science without identifying itself functionally as well as structurally in biological terms.

Relative to the above-mentioned natural function or business of humanity, we may distinguish three levels or orders of structures exhibited by the individual: first, the germ plasm, which is potentially immortal; second, the autonomic apparatus, which is

instrumental to the germ plasm; and third, the projicient or "voluntary" apparatus, which is instrumental to the autonomic apparatus and thence to the germ plasm. If the germ plasm appear to be instrumental to the autonomic apparatus, and this in turn to be instrumental to the projicient apparatus, this may clearly be interpreted as a case of that circular or reciprocal instrumentalism in which the user of tools must be instrumental to his own tools, by way of keeping them in working order.

The expression, "projicient apparatus," I have appropriated from Dr. Edward J. Kempf's *Autonomic Functions and the Personality* (Nervous and Mental Disease Publishing Co., New York, 1921). It refers to that portion of the organism's reactive apparatus which is immediately involved in projecting itself towards and away from objects of its environment. "In the higher animals and man an autonomic or affect-producing sensori-motor system exists which uses a projicient sensori-motor system as a means to project and keep itself in contact with the environment. . . . The projicient sensori-motor apparatus, it appears from the nature of postural tonus and kinesthetic imagery, is, in a sense, the thinking apparatus of the organism" (p. 1).

Adaptations to environment, and social adaptation of individuals to each other, which are naturally-rationally effected with reference to conservation, improvement, and consequent promise of further continuance of the species life, are effected directly through action of the projicient apparatus, which includes the speech apparatus. Yet this projicient apparatus may be stepped up, its potential may be heightened, for the meeting of emergencies, by action of the autonomic apparatus—an aspect of the circular instrumentalism just noted.

As a motilely adaptive portion of the organism, the projicient apparatus may be differentiated into three serially arranged classes of genetically and functionally related structures—effectors, connectors, and receptors, together with the skeleton as a scaffolding and serving as a system of levers for locomotion, manipulation, maintenance of posture, etc. I mention the effectors first in this series, because they serve as the prime motivators as well as the effectors of adaptation. Their cells may be likened to traps that

are more or less constantly set, straining for nervous currents to come out and effect their release into action, and thence straining for stimuli as trips to the receptors, thus to release the neural tracts into action. Analogies of autonomously renewed powder charges to be set off, and of autonomously recharged Leyden jars, may also be employed for purposes of suggesting their nature, with correlative analogies for suggesting the nature and working of the nervous system and of the receptors. Our traditional conceptions and terminology employed here—for example, the terms stimulus and effector—are mechanicalistic in character, implying the assumption that organismic action is of the nature of a push or drive from behind. These are quite certainly inadequate and even erroneous as means for comprehending and describing organismic action, and hence they keep us wrongly oriented for purposes of apposite experimentation and understanding of the case. For one thing, our ingrained Euclideanism perhaps acts as a stumbling block here. We fail to conceive clearly of the organism as being processal in character, an on-going, spatio-temporal thing. Whence we tend to mistake spatial for temporal precedence, or to confuse them. I am suggesting that, functionally, the "forward" end of a reaction apparatus—namely, the effector—is actually the precedent factor leading to action. "Animal pull" would seem to be a more apposite expression than "animal drive."

Just what determines that one and another pattern of effectors shall be in a process of tension conspicuously demanding release into action, is perhaps not yet quite clear. In cases of infants and young animals, these are primarily determined by autonomic tensions, which may even be induced by chemical demands of the blood supply. These autonomic tensions somehow induce tensions in such patterns of the projicient apparatus as have become organized in instrumental relations to consummatory demands of the organism, represented by these tensions of the autonomic apparatus. Thus hunger, as the initial end of an act, is a case of preliminary or "anticipatory" tensions of the ingestive and digestive apparatuses; and the later action of these upon their appropriate sorts of object will constitute the consummatory end

of the act. The same description, with proper modifications, applies to all the life needs of human adults, and to their consummations or satisfactions. In other cases, we have rationally —substitutionally—set up consummatory objectives of perhaps culturally higher order, the structures for specifically exhibiting these consummations being now anticipatorily in process of tension. For example, with the approach of spring, the young baseball player's skill-apparatuses develop tensions that will later be released into overt activity, and will require arduous but joyful practice instrumental to hoped-for success. But whether it be a matter of demand for food or for a full baseball-playing situation as consummatory object, it is the organism as a whole which exhibits the demand, and which will enjoy the consummation. The consummatory apparatus here constitutes only the portion that is most conspicuously in evidence, as being instrumental to the whole. Through this, it is, it would seem, that the organism as a whole makes its demands upon the projicient apparatus and sets this in patterned tensions relative to the satisfaction of organismic needs, and thence relative to the consummatory objects that will satisfy these needs, and thence relative to the habits, skills, knowledges and objective means that will be instrumental to a satisfaction of these needs.

Thus even habit structures and knowledge structures, originally organized as instrumental structures, become consummatory structures, their anticipatory tensions setting the organism to work to find or create consummatory objects upon which they may act and release these tensions. So it is that the activities of common life have become rituals in the life of primitive man, and that skills and knowledges have become abstracted and worked up into games for diversion and relaxation, incidentally giving these organizations exercise and keeping them fit for participation in the more serious affairs of life. If we will but take this time-dimensional view of the organism and its intellectually significant activities, our psychology will gain a far greater semblance of vitality and significance than it generally has today, with its cross-sectional view of its object matters. So much for a brief over-view of the organismic conditions of knowledge.

Regarding the nature of any item of knowledge, we must recognize three temporally serial levels of it: first, that of acquisition or learning; second, that of exhibition in use or action; third, that of possession when not in use or exhibition.

Learning takes place incidentally to action relative to the securing of some desired stimulus, or relative to reaction to such stimulus itself, or to some stimulus intervening between the initial and the consummatory ends of a complete act. At its best, the act that results in learning, or knowledge, is initiated by a patterned set or tension of reactive apparatus, an anticipatory tension of some set of consummatory apparatus, whose full release into action will constitute the consummatory end of the act. Thus it is that the learnings of young children and lower animals start with organismic demands, made through the autonomic apparatus, and conclude with a satisfaction of these demands—a release into action, of the pattern of apparatus that is in process of tension. Similarly, with the young baseball player. Upon a basis of some organismic demand, then, the learning activity best goes on; and the learning consists in an organizing of some pattern of reactive apparatus, as an effect or deposit left by the making of the response. And this organization, as a pattern of the organism's own current processal existence, is a spatio-temporal affair; so that the initial end of the act, as a current semantic substitute for the consummatory end, serves to promote the search for mediating stimuli and to give continuity to the act as a whole.

When, as we say, we know a thing, what is it that we know? We never learn or know a thing, "event," or object as such. Rather, all that we ever learn or know is to react, and how to react, to given stimulus patterns, as represented in experience by patterns of "sensation." Even knowing that some receptor has been stimulated, or that some pattern of sensation is a constituent of experience, consists in responses made to such facts. Knowing always consists in responses, and a response never is, or gives, a literal exhibition or reproduction of its own stimulus. Next, we must note that our knowledge of (about) a thing is likely to consist of many patterns of reaction to it—as focusing of the

eyes, manipulation, supporting postural reactions, descriptive speech reactions, and so on. We may have learned these reactions at different times, and have practiced them under different conditions; but integrated action of their several apparatuses, as in a percept or concept, is likely to proceed more or less simultaneously. And finally, there is the "naming" or denotative speech reaction; our knowledge of an object never seems to be complete or satisfactory until we have learned or personally developed this. This statement suggests that the proper order of experience in learning a vocabulary or word-map should proceed in a definite order: first, contact and experience with things; and later, the verbal denotation of the things. This has been the order of language development in the race, and should be for the individual; at least through his early years. And when the individual learns the denotative reaction, or name, he should *not* be taught that the thing *is* the name; or the name, the thing. Rather, he should learn that the name *is not* the thing, just as he would not identify a map with the territory semantically represented by it.

But not every reaction is necessarily a constituent of knowledge. It is such only when it is a semantic reaction—when it means an object as a previous, or a present, or a potentially future source of particularly patterned stimulation or sensation. In case the object (event) itself serves as the source of stimulation, the semantic reaction is called a percept. In case a semantic reaction meaning an absent or potentially perceptual object is made in response to perception of some semantic substitute for that object, then the reaction is of the kind that we call an idea or concept.

We may illustrate all this by describing a percept or a concept as a reaction rosette. The several "petals" of the rosette represent the various observational and adaptive reactions that we have learned to make to the perceptual or conceptual object. These will be few or many, and will vary in pattern according to our experience with the object, and with others similar in kind or pattern. The name or verbal denotation of the object is relatively homologous from person to person of a given speech culture,

as being socially inculcated for purposes of ready communication. This denotative reaction we may describe as the stem or "handle" of our reaction rosette. It is the response by which we "handle" our concepts, both socially and in personal thinking. When we use our words really attentively or thoughtfully, more or less of these petals of our reaction rosette are in process, though we may not be aware of the fact. In cases of familiar, habitized conceptual reactions, the word itself, as being closely related, semantically, to these petals, is likely to carry quite the entire burden of our current thought about the object of its reference.

This pictorial representation of perceptual and conceptual reactions must not of course be taken too literally. First, it represents them in a cross-sectional, Euclidean manner, and in this respect is wholly misrepresentative, as they should be understood in processal, spatio-temporal terms. Second, it represents each adaptive reaction as being spatially distinct from the other, whereas there may be a large portion of the organism functionally common to even many of them—for example, in the postural apparatus supporting the more specific reactions relative to the stimulus object itself. Yet with these qualifications, I believe the picture is highly illustrative; and it represents an order or level in the conceptual series that Korzybski has overlooked, in the construction of his "Structural Differential." His "object" is not the percept, or semantic reaction; nor is the name, or speech reaction. His "object" constitutes what may be called the sense object, or "phenomenal object"—the sensation pattern generated by stimulus contact with the thing or "event." It is identifiable as the reaction of the sensory apparatus involved, which serves as the cue for perceptual adaptation and thence for overt organismic adaptation; and it is just as truly natural or physical as is any other fact of nature. The semantic reaction that as percept or concept serves as a term of thought, is both spatially and temporally intermediate between the sense object and the word or denotative reaction. On the other hand, Korzybski is everlastingly right in insisting that the "object" *is not* the thing or "event" which serves as its occasion. This, he says, is ineffable or unspeakable—which is another way of saying that a response is

not the same as its trip or stimulus, nor a literal reproduction of it. So the successive orders of events culminating in knowledge of an "event" are designable, in order, as organismic tension, "event," sense object, percept, concept, word or name, description, definition—of name or description, definition of definition, *ad indefinitum*. If we were to consider this series genetically, we should find that the sense object developed from mere stimulation, and the percept from overt adaptation; as the organism became able to refrain from immediately overt reaction and attend to the stimulus, in the interest of improved adaptation, these two orders developed into sense object and percept, respectively.

Two further observations are in order regarding the concept as a reaction. First, we often speak, loosely, of making "the same" reaction, and of "repeating" reactions, and of "having the same" idea from time to time. In a processal world, this is obviously impossible. We may make successive reactions of the same pattern, but it is not even the same "we" that make these serial reactions. Second, our old mind-body dualism is so deeply ingrained in our language and in our ways of using it, that we habitually speak of "making," effecting, or executing reactions—as I have just done, as if our reactions were somehow differentiable from us who "make" them. Our current ability to think of our world and of ourselves in processal terms should serve in time to eliminate this wholly inapposite mode of speech and thought—or thoughtlessness. Any reaction that we "make," as we say, is a spatio-temporal change in the on-going patterns of the spatio-temporal things that we are. This is obvious in the case of overt reaction, even the winking of an eye. We do not, as Euclidean objects, make reactions that are somehow other than ourselves; we *live* our reactions, perceptual and conceptual, no less than overtly adaptive. Perhaps this is the reason that we cling so tenaciously to our ideas and beliefs; to be required to give them up would be like—in fact, it would be—yielding a part of our very selves. Plainly, we cannot give them up or throw them off. What we have to do, in many cases, is to develop new or corrected semantic reactions, to supplant the old or inapposite ones, which

will then no longer constitute a part of our personal thinking. Yet they may still help us in our social adaptations, as they help us to understand others, who still live the old patterns of thought; and so we may help these others to develop new and supposedly better patterns, to supplant their old ones.

Having thus given, in organismic terms, a brief over-view of the nature and content of thought, we may now consider for a moment its logical aspect. The logical relation of a concept to its object of reference is that of serving as a semantic substitute for it. And the effective relations between concepts themselves are those of serving as reflections, semantic substitutes, or map relations of the processal relations between their objective references. And the "name" of an objective reference, as being reactionally related to all the variously patterned reactions that this object has evoked from us, serves as an organizing center for binding all these together into a concept, and thence as a semantic substitute for the concept itself, and thence for the sense object, and thence for the stimulus "event."

Thus our rational repertory may be described, genetically, as a series of maps of maps of maps of things, actions, and relations to which knowledge refers. And so it is that scientific endeavor may be described as intellectual map-making—the development of such responses, and of such an organization of structures for exhibiting these, that they may serve as semantic substitutes for the things observed. The physicist's "objects," such as atoms and electrons, serve him as preliminary, trial maps, to guide him in more intensive exploration, whether to substantiate and fill them in or to aid in the construction of more apposite ones. As in all other cases of knowledge, these are constituted of reactions; they *are not* the things mapped. Even such indefinitesimally minute entities as those with which he is trying to deal, never appear in the reactions that he develops by reacting to data concerning them.

In cases of historical inquiry, our evidence, no matter from what era it has survived or in what era it was deposited, is constituted of still current facts or data. These are describable as physical things, whose processal patterns have been so constant

as to be essentially homologous throughout the interim, and which are comparable to data of current matters with which we are familiar. Upon a basis of conceptual reactions derived from experience with current, less isolated and less fragmentary facts, we use these processally surviving data of past events with which to construct concepts as nearly as possible apposite to the totalic pattern of facts for which they serve as data. It is common to say that we can understand the present only in the light of a knowledge of the past. On the contrary, it is only out of a knowledge of the present that, with current data of any past era, we can frame any sort of adequate conception of that era. Historical knowledge in the getting must always be genetic or regressive—from the better known present and near to the distant and less well known past; though we may make our account of it follow the supposed order of events, thus determined.

In cases of anticipatory knowledge of the future, we have recourse to current conceptual maps, constructed with reference to a feared or a desired future; and then we either wait to see whether these maps prove to be apposite, or set about it to keep them from being fulfilled, or to make the future apposite to them. And then there arises the question of how apposite to that conjectured future reality is our present concept, and how appropriate are our mediating concepts and other instrumentalities, and our uses of them, in relation to that conjectured future.

We thus have two fundamentally pragmatic criteria of "truth" —one to be employed in critical appraisal of our percepts and concepts of situations genetic to action, and those of ends, and of means *as objects*; the other, in critical evaluation of means *as means*, in relation to ends.

The first of these is the genetic test, made by observing whether the perceptual and consequent conceptual reaction is apposite to its generative or stimulus situation. This criterion I call that of "Appositeness." It is a pragmatic version of the criterion historically called "Correspondence," and it does not imply that a truthful concept must be "like" its object of reference, as the Theory of Correspondence is likely to do. It is the one that Dewey has used so effectively, in his *Quest for Certainty*. It is

the one which we always employ when we ask whether a statement of fact is "true." What we are asking, in such a case, is whether the statement, in its medium, is an accurate reflection, or presents an apposite map, of the situation to which it semantically refers. If we state the case in reference to a complete act of purposive or rational adaptation, then this criterion is one that we should first employ in relation to the terminal end of the act, or even to some further objective, to which this will serve as means. Here the question is whether or not that end is desirable and possible of attainment, and whether our concept of the consummatory object itself is apposite to potential fact. Second, we employ this criterion in evaluating our percept or concept of the current situation; for our determination of this will serve as a means to our end, as this, together with our concept of the end, will determine what further means must be employed. And third, we must employ the criterion of Appositeness in appraisal of our perception and conception of these intermediate means. We are generally so familiar with these means that we use the criterion here quite automatically. In relation to our ends, we are generally so unspecific and so myopic in specifying these, that our purposive behavior is little better than mere opportunism. It is in relation to diagnosis of our problematic situations that we most consciously and consistently apply the criterion. But its use is no less desirable and necessary in relation to our determination of both ends and means.

Our second criterion I call that of "Appropriateness." This is an instrumental criterion. It has application to our concepts and external things that we use *as means* for the attainment of our objectives. For example, the proper objective of a physician called to see a patient is that of beholding at the end of his ministrations a well person. He has learned sets of concepts and habits respectively adapted for treatment of patients afflicted with various ailments. In relation to any patient, his question is, "Which of these will be appropriately instrumental to a renewal of health?" In our current personal and social thought and conduct, there is the question of whether or not these serve rather as helps or as detriments to such racial welfare as will make

for support and improvement of the quality of the species life and hence promise an ever and ever further continuance of the species life. Are our concepts—and their references—appropriate instrumentalities for the realization of conceived ends, and do we use them appropriately to our ends? The test consists in putting our concepts to work as semantic substitutes for the objective means that we shall use, and then putting the things themselves to work and seeing whether the ends are realized as conceptually anticipated.

It is customary to say that this test of truth is a test of control. Thus science is always seeking to "control" things and processes.[5] Contrary to this view of things, developed out of machine-age patterns of thinking, I am suggesting that control, as sought by science, is properly the pragmatic test of Appropriateness. Provided the concepts, as semantic substitutes for their references, meet this test, they are then available for use in technology—provided the technological objectives and practices are biologically justifiable.

A third criterion of truth, often used, is that of Coherence. The advocate of this criterion assumes that the logical coherence of concepts in a system is itself an evidence of their truth value. But only in case the fundamental postulates are tenable, as being semantically apposite to an intended reality, can this be a valid criterion. Facts will have the last word, however coherent our concepts with each other. In other words, only in a logical system of concepts in semantic apposition as a map of a coherent reality, is coherence itself an evidence of truth. Even the criterion of Coherence is subject to the criterion of Appositeness.

[5] This reference to "control" does not apply to the endeavor of the scientist to control the variables in his situation, but to the fact that his hypotheses and their references serve as means for bringing about desired consequences. Here the question is whether they are *appropriate* as means for such purposes. The question is answered by putting them to work and observing whether in fact the anticipated consequences do follow—provided no significant variables in the situation have been overlooked. Solution of the problem of error involves the use of both of these criteria. There are errors of both appositeness and appropriateness; but so far as knowledge as such is concerned, the final question is whether or not it is apposite, in its medium, or its generative situation, so that it can be used as a semantic substitute for that situation.

It would be interesting and profitable to consider the appositeness, in a processal world, of the three historical Laws of Thought —the Principles of Identity, Contradiction, and the Excluded Middle. But these do not necessarily come within the limits of my subject. Besides, any adequate discussion of them would require more time and space than I have at my command here.

In conclusion, I may remind you that I have represented the knowledge enterprise as exhibiting three serial levels—those of learning, of exhibition or use, and of possession. I have also indicated how the learning or acquisition goes on, and have described knowledge in exhibition as a matter of semantic reactions. As for knowledge as a possession, when we are not using it, this—as already implied—may be said to consist in organizations of structures whose integrated action, directed towards their "events" or semantic originals, constitutes knowledge in exhibition. In all this, I have not been guilty of reducing any one sort of thing to any other sort, and I have not explained anything away. I have no problem of getting the knowing Humpty Dumpty together again, for I have not taken him apart. The point of my whole paper has been to *suggest*, in a cursory and elementary way, that knowledge is an affair of the organism as a whole, in relation to its environment, and not of the brain and nervous system alone, or of sensation complexes; and that its proper function is that of serving biological ends. If I have succeeded to this extent, however much correction in detail may be required, then perhaps some few of us may soon discern the way of proceeding to the correcting, or even to an experimental validating, of the view here presented, and of turning the knowledge enterprise more specifically to its proper function. Or possibly my argument and the assumptions underlying it are all awry and require neither argument nor experimentation to refute them; in which case I must apologize for imposing them upon you.

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DISCUSSION (CORRESPONDENCE)

In this section we shall publish brief communications commenting on published papers or bringing up special points of interest which can be discussed by correspondence.

QUANTUM MECHANICS, HALDANE, AND LEIBNIZ

Dear Sir:

There has recently appeared in this journal a very interesting paper by Professor J. B. S. Haldane, entitled, "Quantum Mechanics as a Basis for Philosophy."[1] The paper might indeed be called, "Leibniz as a basis for Quantum Mechanics." The name of Leibniz does not occur once in the article, yet it is permeated with his spirit, and is just such a document as an orthodox Leibnizian might write should he be confronted with the development of mathematics and experimental science in the two centuries since the death of the master.

It may seem a gross an achronism to speak of Leibniz and of quantum mechanics in one breath, but this anachronism disappears when we reflect on the historical background of Leibniz. Our classical physics, the physics of our fathers and our text-books, is dominated by Newton and his dynamics of particles. We had come to regard this science as physics *par excellence*, and to demand a mechanical model for our optics, electrical theory, etc. This dictatorship of mechanics has only given way to a more democratic rule in the physics of our present generation. Thus we are inclined to forget that this dictatorship is not coeval with physics itself, and that there were heros before Newton. Among the pre-Newtonian physicists the greatest was Huyghens, the prophet of wave optics and the master of Leibniz *qua* physicist.

The present state of physics represents a Hegelian synthesis between Newton's thesis of particle and Huyghens' antithesis of wave. Modern optics supplants the pure wave theory of Clerk-Maxwell by a theory in which the particle aspects of light are more strongly emphasized. In the field of mechanics, on the other hand, the well-intrenched particle

[1] Philosophy of Science 1, 78–98, 1934.

theory has been superseded by one recognizing the wave aspects of matter. For the first time since Newton, wave and particle, field theory and a theory of collisions, share all branches of physics more or less evenly.

It is traditional to represent Leibniz' theory of monads as a pluralistic spiritualism. If we divest Leibniz' language of its protective layer of orthodox Christian phraseology, we shall see that it might as well be called a pluralistic materialism. Only a small part of the monads are souls, the greater part being naked monads gifted with apperception rather than perception or consciousness. They mirror the universe, but do not integrate this activity of mirroring into a self-conscious consciousness. This mirroring is best to be understood as a parallelism, incomplete it is true, between the inner organization of the monad and the organization of the world as a whole. The structure of the microcosm runs parallel to that of the macrocosm.

In modern quantum theory, the electron or similar particle is a microcosm. It carries with it a set of proper coördinates embracing the entire universe. In terms of these coördinates we may express everything happening everywhere and "everywhen," but the picture we thus obtain is blurred because of our ignoration of the proper coordinates pertaining to all other particles, and we have no unique way, as far as physics has gone at present, of translating statements concerning the microcosm into statements concerning the macrocosm. The more organized a system of elementary particles is, the less "naked" its electrons and other constituent elements, the better it mirrors the universe, and the better can we read back from our partial system to the universe at large.

Haldane assimilates the living organisms of the world to the particles of physics. He reduces the apparent randomness of their action to the indeterminacy of quantum theory and the ability which they show to make a choice on the basis of a representation of the future, to the ability of an electron to cross a potential barrier because of a lower potential on the other side of that barrier. In other words, although Haldane calls his system a materialism, and just *because* he calls it a materialism he reduces the living organism and the soul to something of the same nature as the proton or electron. I can see no essential difference between the materialism which includes soul as a complicated type of material particle and a spiritualism which includes material particles as a primitive type of soul.

With all these similarities between Haldane and Leibniz, there are

important differences which cast a curious light on the changes both of physical and biological science since Leibniz' time. Haldane and Leibniz treat the problem of individuality somewhat differently. For Leibniz the monad is a completely closed entity over all eternity. It may rise to the rank of a soul or be degraded to that of a naked monad, but it never loses its identity or fuses it with that of another monad. On the other hand in deference to the exigencies of modern biology and modern physics, Haldane does not make the individual identity of his particles or living beings absolute. The electrons of present-day physics do not retain a traceable identity after a collision, nor is it possible to trace the identity of a living organism back of the conjugation of germ cells in which it has its origin. These two types of indeterminate identity known to modern science, are identified by Haldane in an extremely interesting speculation. On the other hand, at the time of Leibniz, neither problem had arisen. No one doubted that the identity of two particles could be traced beyond their collision and the prevailing spermatist theory of heredity and conception made the individual find its sole origin in a single spermatozoon, to which at that time was supposed to have an existence continuable back to the Creation. Nevertheless, Leibniz' great principle of the identity of indiscernibles, to which he appeals in connection with the perfect individuality of the monad, is retained in the modern view. Since we may permute two electrons in such a way that they will be indiscernible from each other, we now say that two electrons cannot have complete separable individualities, but must be no more than two aspects of a complex containing two or more electrons. The modern philosopher is more subtle than Leibniz in distinguishing between a unity of indiscernibility and a multiplicity arrived at by counting. This unity in plurality is perfectly familiar to the mathematician: for example, the Möbius strip is locally endowed with distinguishable sides, but as a whole has only one side.

The other important issue on which Haldane and Leibniz take opposite sides concerns the nature of causality. With Leibniz, causality is still an unbroken chain, with no place for chance, while Haldane takes the modern standpoint according to which chance is a category prior to causality. It is thus all the more remarkable that the Leibnizian principle of sufficient reason is a direct ancestor of the mathematical methods of quantum mechanics. The principle of sufficient reason asserts that this world differs from the other possible self-consistent worlds by embodying some greatest perfection—in other words, by satisfying some principle of minimization. This must have been sug-

gested to Leibniz by his knowledge of optics and his acquintance with the principle of Fermat, in accordance with which rays of light minimize the time of passage between two near points. Leibniz calls this principle the principle of optimism, and in this form it may readily serve as a basis for a rather fatuous contentment with the world as it is. Voltaire ridiculed this contentment on the part of the Leibnizian Maupertuis, but the fact remains that Maupertuis' application of Leibnizian method to the expression of the laws of Newtonian mechanics as the minimization principle of least action, has proved to be anything but fatuous. It must always be considered as a mark of the greatness of Leibniz, that his idea of representing the laws of physics by principles of minimization is the source of the mechanics of Lagrange and Hamilton, from whom the line of inheritance to Heisenberg and Schrödinger is clear and direct.

Again, Leibniz may be regarded as the forerunner of field theory.— For Leibniz, the world is a plenum, and there is no action at a distance. A monad can only mirror a distant monad through the intervention of intermediate monads. Thus Leibnizian physics only awaits the introduction of the partial derivative to become a field physics. This step is taken in the writings of Leibniz' immediate disciples, the Bernoullis.

Leibniz clairvoyance into the future is explicable only on the basis of his universal information, universal interests, and universal genius. It is scarcely an accident that a scholar of the broad knowledge and interests of Professor Haldane takes up what is essentially a Leibnizian position. A philosophy with sufficient vitality to appeal to first rate scholars two centuries apart is surely worth more consideration than that generally granted to it by the intellectual public.

N. Wiener.

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THE PRINCIPLE OF INDUCTION

Dear Sir:

Since methodology is fundamental to all science and logic, may I be permitted to make a remark upon the methodology of Professor Feigl's very interesting paper, The Logical Character of the Principle of Induction?[1] This remark is not intended in any sense as derogatory to what

[1] This Journal, Vol. 1, p. 20 f. (1934).

is an excellent paper; it is merely an attempt to assist in solving the important issue he has raised.

He founds his paper on the statement that the unsatisfactory state of the problem of induction is due to philosophers' preoccupation with the problem of the validity of induction, whereas the more fundamental question concerning the meaning of the principle of induction, to which he addresses himself, has been rather neglected. (p. 22)

But are these two problems really distinct? For the problem of the meaning of induction (in which the chief difficulty, according to Prof. Feigl, lies in the interpretation of the problem of probability) is, stated more exactly, the problem of the *correct* interpretation of these concepts. There are many interpretations of induction and of probability; this paper is itself an attempt, not to define previously undefined concepts, but to discover their *correct* interpretation. Then he must logically obtain, previous to securing any 'correct' interpretation, some adequate method of determining correctness. Since, now, as he says, "Induction is the indispensable foundation of all factual science" (p. 21) some theory of induction becomes the presupposition of obtaining any 'correct' account of induction. Then previous logicians, who concentrated upon the problem of the validity of induction, were quite right in their procedure. Instead, therefore, of the problems of the validity and meaning of induction or probability constituting two problems, they are but two sides of the same problem. The validity and the correct meaning of an ultimate concept such as induction must be determined at the same time by the same procedure.

In conclusion, may I suggest that the solution of the problem of induction may be found in a direction indicated by Prof. Feigl, though not worked out by him? He admits that "The more advanced factual sciences do not actually proceed by inductive generalization. Their method consists rather in the construction of hypothetico-deductive systems." (p. 21) It is then possible that what has been called 'induction' is not the true foundation of the more advanced and therefore more adequately established sciences. 'Induction' may be a procedure adopted by the less advanced sciences because of their own inadequacy. The fact then that induction "can be proved certain neither on logical grounds nor on the basis of its own success" (p. 21) does not prevent finding an adequate logical foundation for the factual sciences in the hypothetico-deductive method. It is this consequence of generally accepted logical views that I have worked out in my theory of 'rational induction,' to which the distinction between what has been called

'induction' (which I style 'empirical induction') and the hypothetico-deductive method (which I style 'rational induction') is fundamental. It may then be true that scientists have really been using a logically correct method; logicians have merely failed to recognize its precise nature and the grounds of its correctness.

HOMER H. DUBS.

Marshall College,
Huntington, W. Va.

Dear Sir:

In replying to Professor Dubs' gratifying remarks I should like to elucidate a few important points of my pragmatic view of induction.

I quite agree with Prof. Dubs' contention that the problems of the meaning and the validity of induction "are but two sides of the same question." My purpose in differentiating the two "sides" was only to point out that almost all previous formulations of the principle of induction gave it such a meaning that its validity could not be considered anything but *inductive*.—Also Prof. Dubs' own solution of the problem (H. H. Dubs, Rational Induction, The University of Chicago Press, 1930, pp. 450 to 457) in which he substitutes for the traditional postulate of the uniformity of nature his "principle of the eternity of truth" suffers from this defect. For as he admits (ibid., p. 453) "unfortunately for philosophy and science it must be confessed that at present the permanence of truth is merely an empirical statement. We are merely able to say that what truth has been discovered in the past is still true; we cannot say *why* it has remained true, nor be absolutely sure that it will continue to remain so, since no merely empirical law can attain to certainty. Consequently we have to confess that our armor of defense against skepticism is not complete, for this empirical element is mixed in with all Knowledge." This passage shows clearly that Prof. Dubs' claim of infallibility for the method of "rational induction" (i.e. the hypothetico-deductive method) is not seriously upheld even by himself. His attempts to demonstrate the certainty of rational induction (ibid.; chapter IX, X) and to refute Hume's skepticism (p. 454 f) rest on rather questionable dialectical arguments and on the promise of the future discovery of a metaphysical proof of the "eternity of truth." Now, supposing such a proof could be given, it would have to be based on premises which, by logical necessity, would have to contain *universal* propositions. But, even disregarding my antimetaphysical convictions, I find it plainly

inconceivable that any metaphysical method could validate most general factual propositions without recourse to empirical induction.

It was just this rather obvious logical situation which made me determined to reject any of the formulations of the induction principle which express a factual meaning, i.e. make a statement concerning the characteristics of the world. In my opinion there can be no guaranty for the validity of such and other generalizations, be they simple enumerative inductions or hypotheses of the more advanced scientific type. At any stage of scientific progress (as we know it) there will be outstanding premises, from which the more specific statements can be derived, with—indeed—(*deductive*) certainty; but those premises in themselves are *assumptions*, ever ready for revision, valid only "until further notice."

The principle underlying all induction can therefore have only *pragmatic* significance. Its meaning is not factual but "motivational" (directive); it simply tells us "to go ahead" with our generalizations from past experience and with the establishing of hypotheses-systems. There is no proof that these procedures, even if conducted as carefully as they may, will lead to the success aimed at, i.e. the truth of predictions. But the principle is nonetheless more than an arbitrary prescription and it is certainly not a categorical imperative. To use classical terms, its nature is best expressed as the *hypothetical imperative: If you intend to predict correctly use the method of simplest generalization on the basis of as broad an experiential* (*observational, experimental, statistical, etc.*) *background as you can secure.* This imperative, however, does not and cannot promise the desired success. Its only claim is: If you desire to proceed according to a *method*, i.e. if you are not interested in capriciously guessing, gambling or mystically intuiting the future (more generally the yet unknown, but knowable) then you have *no choice.* Any other "method" like expecting systematic deviations from past regularities or not employing all available experience) would be "madness" because it would not in any typical sense be distinguished from an indefinite number (a continuum, virtually) of equally arbitrary "methods." (The old principle of sufficient reason construed as an operational maxim.) The *justification* of the inductive procedure lies precisely in this uniqueness (extremum character) of the only rational method capable (but not guaranteed) of success. This then is the small step we can go beyond Hume's view of the matter. The step may seem too small to those who still cherish hopes for a better vindication of induction. But those who acknowledge the very simple fact that we don't know what we really don't *know* will be satisfied with having

at least a rational way of taking a chance of knowing what we don't know.

I hope that these remarks will suffice to make clear why I thought it so important to formulate the principle of induction in such a way that its meaning would express the basic regulative maxim of factual knowledge. If, in the analysis that led me to my result, I have made use of induction, such as in taking it for granted that the essential features of human knowledge will remain the same in the future as they have been in the past, I would reject the charge of vicious circularity. I was not forced to use some theory of induction as a *presupposition* of my account of induction. And this for the simple reason that my aim as well as my method were analytic and not inductive. I did not try to establish the certainty or probability of inductive generalization—this I had shown to be an illusory problem—but my goal was merely to analyze factual knowledge and to disclose one of its most fundamental rules of procedure.

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FLEXIBILITY OF SCIENTIFIC TRUTH

Sir:

In the last issue of PHILOSOPHY OF SCIENCE[1] Mr. F. Zwicky has reiterated his announcement of what he calls the "Principle of the Flexibility of Scientific Truth," and has added a rejoinder to my criticism[2] of his previous proposal.

Were it not for Mr. Zwicky's open challenge I should not make this reply (my last), for I feel that the arguments of my former note have not been touched by his latest remarks. The only points I wish to make are these:

(1) Whether Mr. Zwicky attempts to establish the validity of his suggestion by reference to many-valued logics or not has nothing to do with my argument; this argument concerned the fact that flexibility of truth is incompatible with *every* type of logic hitherto proposed.

(2) If Mr. Zwicky intends, by establishing his peculiar "reference system of scientific truth" (ref. 1, p. 354) to set up a concept of scientific

[1] F. Zwicky, Philosophy of Science, Vol. 1, 353, 1934.
[2] H. Margenau, Philosophy of Science, Vol. 1, 118, 1934.

truth different from that familiar to logicians I am unable to follow him. Needless diversification of concepts seems undesirable. From the recognized logical point of view there is an immense class of true propositions, such as: Remarks on the "principle of the flexibility of scientific truth" are printed on pages 354 and 355 of vol. 1 of PHILOSOPHY OF SCIENCE.

(3) Mr. Zwicky's proposal may be heuristically very useful, but it is neither new nor a principle of reasoning. In fact, universal doubt has been proclaimed so often that it has become proverbial. And, after all, let us not forget:

"Douter de tout et tout croire,
ce sont deux solutions également faciles."

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REVIEWS AND NOTES

An Introduction to Logic and Scientific Method, by Morris R. Cohen and Ernest Nagel. New York: Harcourt, Brace and Co. 1934. xii, + 467 pp.

Here is an elementary text that has caught up with the times. It moves in the new circle of ideas. It sets the old beside the newer points of view and lets the one criticise the other. It treats not only logic as such, but of the rôle of logic in the mechanical and statistical sciences, in history, in art and in morals. It is encyclopaedic and at the same time it is competent. It is a book to be welcomed. And it is a book that will be used.

Some of the analysis is very well done indeed, more particularly where it undertakes to remove the common misunderstandings of beginners. This applies in chapter IX, in which the historical objections to the syllogism are met in the section on "The paradox of inference," in which Mill's arguments are answered in the section "Is the syllogism a petitio?" and again where the discussion on "The laws of thought" might be summed up in a paraphrase of Russell's phrase: The remarkable thing about Boole's "Laws of Thought" is the fact that nobody had ever thought of thinking that way before. It may be added that certain criticism of certain pragmatists is (perhaps properly) ignored, though Mr. Schiller on p. 438, 457, is presented as a case for student experimentation and dissection.

In commending the directness and forthrightness of the analysis, attention might be called to the criticism of Mill's canons of induction, which is novel and original. The reviewer is trusting his memory, but he thinks that it was Charles Pierce who said that it was significant that the examples from science, to which the reader was invited to apply the canons, and which appeared in the first edition of Mill's Logic, were omitted in the second. Presumably because they did not apply. Our authors come to much the same conclusion.

We note an occasional lack of strict expression, as on page 144: "In

the Lobachevskian geometry this (Euclid's postulate 5) is replaced by the assumption that through a point outside of a line more than one parallel can be drawn to it." Lobatchevsky commonly begins with the assumption of a variable angle of parallelism, from which it *follows* that through a point outside a line there is double pencil of lines that do not intersect the given line and that *two* of these are parallel or asymptotic to the given line, whereas the others diverge from it.

Or, again, on the same page: "From this assumption . . . a host of theorems may be obtained, some of which are identical with the theorems of Euclid, while others are contradictories of these." More accurately: In verbal contradiction with these. *Verbal* because the terms in the two systems are differently defined. The contradiction is only apparent. A very simple representation of a finite solid (Riemannian) space could have been given by introducing the notion of non-coplaner parallels. It would have been helpful too to have given another simple non-Euclidean parallel construction.

And speaking of opportunities overlooked, on page 145–6, where the reader's doubts concerning the consistency of non-Euclidean geometry are met by showing the consistency of a miniature system, the section should surely end by a brief reference to the work of Beltrami, as otherwise the reader's doubts are not removed.

In another context (page 79) a simple proof of the *necessity* and *sufficiency* of axioms 1–5 might have been given. Thus, they are sufficient because they declare all the invalid moods to be invalid. They are all necessary because we can point to at least one example that falls uniquely under each rule. III (any figure) is declared invalid only by axiom 1, AAA (fourth fig.) only by 2, EEE (any fig.) only by 3, AEI (any fig.) only by 4 and AAO (fourth fig.) only by 5. Therefore none of the axioms—better called rules, since they are formulations based on a complete induction—can be dispensed with.

There is one statement in the book which the reviewer would like to challenge, as he has challenged it many times before: "We do not believe that there is any non-Aristotelian logic in the sense in which there is a non-Euclidean geometry, that is, a system of logic in which the contraries of the Aristotelian principles of contradiction and excluded middle are assumed to be true, and valid inferences are drawn from them" (page v).

Now in the first place this is quite a false definition of what would constitute a non-Aristotelian logic. A logic to be "other" than the traditional one would *not* have to differ from tradition on these points.

It would have, I think, to fulfill at least three conditions: (1) it would have to exhibit at least some properties which the traditional system did not share, (2) it would have to be applicable to the same world and (3) there would have to exist transformation formulas by which the one system could be translated into the other. This last condition realized, the two systems would have exactly the same degree of generality and any claim that the "other" logic is more general than the traditional would thereby be refuted. We will show that these conditions can be represented and the non-Aristotelian system we will select by way of illustration will be the logic of Boole. It has among others the following properties:

Boolean or non-Aristotelian Logic

$$(a \supset b)\cdot(c \supset d)\cdot(ac \supset bd)' = o$$
$$(a \supset a)' = o$$
$$(a \supset b')\cdot(b \supset a')' = o$$
$$(a \supset b)\cdot(b \supset c)\cdot(a \supset c)' = o$$
$$(a \supset b)\cdot(ab' \supset o)' = o$$
$$(a \supset b)'\cdot(ab' \supset o) = o$$
$$(ab \supset b)' = o$$
$$(ab \supset c)\cdot(ac' \supset b')' = o$$
$$(d \supset b)\cdot(ab \supset c)\cdot(ad \supset c)' = o$$

wherein $\supset$ stands for Boolean inclusion, the letters for classes, the prime for negation and o for the zero of classes or of propositions. If the symbol $<$ be used to represent Aristotelian inclusion, then all the properties of the traditional system, viz.

Aristotelian or non-Boolean logic

$$(a < a)' = o$$
$$(a < b')\cdot(b < a')' = o$$
$$(a < b)\cdot(b < c)\cdot(a < c)' = o$$
$$(a < b)\cdot(ab' < o)' = o$$
$$(a < b)\cdot(a < b') = o \quad \text{subalternation}$$
$$(a < b)\cdot(a' < b) = o$$
$$(a < a') = o$$

follow formally and for *all meanings of the terms*, if we assume the logic of Boole to be true, by means of the transformation formula,

$$(a < b) = (a \supset b)\cdot\{(b \supset a) + (a \supset b')'\cdot(b' \supset a)'\}$$

The meaning of zero will be defined in terms of Aristotelian inclusion if we substitute $a = aa'$, $b = a$, in the formula, that is,

$$aa' < a = a \supset aa'$$

so that the zero of classes is any a that satisfies,

$$(aa' < a)' = 0 \text{ or } (a \supset a')' = 0$$

Similarly, assuming the logic of Aristotle to be true, we may prove all the properties of the Boolean system by means of the transformation,

$$(a \supset b) = (a < b) + (aa' < a) + (bb' < b')$$

The two systems are, therefore, intertranslatable and consequently, of exactly the same degree of generality. All algebrists of logic have committed the formal fallacy of identifying the symbols $<$ and $\supset$, that is, the two meanings of inclusion, and have hence concluded that Aristotle's logic breaks down.

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A NEW BUDGET OF PARADOXES

(4) NOTES FROM A CASE-BOOK

We continue the discussion of our classification of paradoxers by exhibiting some cases.

Case 1. Magician, sub-class paladin of the balance. Ph.D. 1905 claims to have discovered a primitive logic whereby he accomplishes the "resolution of all cases of the hitherto unrecognized general problem, the differential coefficient = function." Title of the work: Science {Function}Analysis. Everything is solved on the wing, but no details.

Case 2. Magician, sub-class illusionist. Everything is solved, but painful details, including portrait of the author, who is also author of "Galomalism." The "galom," discovered in 1883, through remarkable "operations on the drawing board such as an architect will make," is nothing else than absolute being, or the constant forceproduct as the essence of the world stuff. The first galomaniac was Boyle, but he was not aware of it, nor did he quite realize that chemical cold times chemical heat also gives a constant. Every substance has "spantoms" which on being chipped or shaved off give color (Berkeleyans beware!). *Materia* has its *pateria*. Lightning is negative electricity. There you have a chance also to make the acquaintance of the *chemograph*, the *temperatometer*, *chemicature*, and the *zeron*, or the large group of organic substances at or near the zero of nature or the point of death. The climax of the work seems to be *Nirvanology*, or the science of death, which proves the existence of ghosts. These reside in the shadow or "tail of the earth," of which there is a diagram, oriented with regard to the latitude of New York. A large "G" stands out, and turns out to be the ghostly home of Galileo! No one has claimed a $1000 prize offered by the author for a disproof.

Case 3. Poisoner, sub-class annihilator. Title: Force and Matter, or the Evolution of Forces, "written for the masses of the people." The main argument, as of 1908, is that ether is the force of matter or the force that acts on matter. Disproves everything, including the

famous "the earth moves up to meet the apple." Heat is not molecular motion: "In heating a mass of iron, for instance, a cube or a sphere, if it be placed within a furnace where the heat entirely surrounds it, it is evident that the molecules must move outward, for the heat produces expansion, while the heat passes in towards the center by conduction. Thus the heat and the molecular motion move in opposite directions, and are two different and distinct entities." Finds the longitudinal air-wave (sound) to be a plain fraud, and clinches the argument with irony. "Again," he says, "a locust by the stridulations of its legs, as it makes a noise, moves *5000 millions of tons of air*, since it can be heard at the distance of a mile in every direction." Galileo and Copernicus were also unrecognized.

Case 4. Phantom-Crier, sub-class alarmist. Karl Dietrich, Berlin 1887. "Protest gegen die moderne Wissenschaft," with a foreword to the Kaiser, and a kind word for Bismarck. Circulated at the German universities.

W. M. M.

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